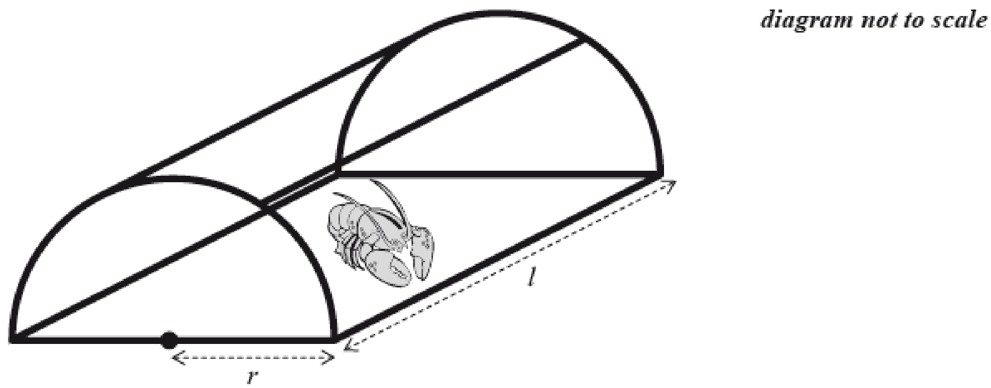
SL Paper 2

A lobster trap is made in the shape of half a cylinder. It is constructed from a steel frame with netting pulled tightly around it. The steel frame consists of a rectangular base, two semicircular ends and two further support rods, as shown in the following diagram.



The semicircular ends each have radius r and the support rods each have length l.
Let T be the total length of steel used in the frame of the lobster trap.

a. Write down an expression for T in terms of r, l and π. [3]

b. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. [3]

Write down an equation for the volume of the lobster trap in terms of r, l and π.

c. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. [2]

Show that $T = (2\pi + 4)r + \frac{6}{\pi r^2}$.

d. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. [3]

Find $\frac{\mathrm{d}T}{\mathrm{d}r}$.

e. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Show that the value of r for which T is a minimum is $0.719\ \mathrm{m}$, correct to three significant figures.

f. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Calculate the value of l for which T is a minimum.

g. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Calculate the minimum value of T.

Markscheme

a. $2\pi r + 4r + 4l$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for $2\pi r$ ("π" must be seen), ***(A1)*** for $4r$, ***(A1)*** for $4l$. Accept equivalent forms. Accept $T = 2\pi r + 4r + 4l$. Award a maximum of ***(A1)(A1)(A0)*** if extra terms are seen.

[3 marks]

b. $0.75 = \frac{\pi r^2 l}{2}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for their formula equated to 0.75, ***(A1)*** for l substituted into volume of cylinder formula, ***(A1)*** for volume of cylinder formula divided by 2.

If "π" not seen in part (a) accept use of 3.14 or greater accuracy. Award a maximum of ***(A1)(A1)(A0)*** if extra terms are seen.

[3 marks]

c. $T = 2\pi r + 4r + 4\left(\frac{1.5}{\pi r^2}\right)$ ***(A1)*(ft)*(A1)***

$= (2\pi + 4)r + \frac{6}{\pi r^2}$ ***(AG)***

Notes: Award ***(A1)*(ft)** for correct rearrangement of their volume formula in part (b) seen, award ***(A1)*** for the correct substituted formula for T. The final line must be seen, with no incorrect working, for this second ***(A1)*** to be awarded.

[2 marks]

d. $\frac{\mathrm{d}T}{\mathrm{d}r} = 2\pi + 4 - \frac{12}{\pi r^3}$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for $2\pi + 4$, ***(A1)*** for $\frac{-12}{\pi}$, ***(A1)*** for r^{-3}.

Accept 10.3 (10.2832...) for $2\pi + 4$, accept $-3.82 - 3.81971\ldots$ for $\frac{-12}{\pi}$. Award a maximum of ***(A1)(A1)(A0)*** if extra terms are seen.

[3 marks]

e. $2\pi + 4 - \frac{12}{\pi r^3} = 0$ **OR** $\frac{\mathrm{d}T}{\mathrm{d}r} = 0$ ***(M1)***

Note: Award ***(M1)*** for setting their derivative equal to zero.

$r = 0.718843\ldots$ **OR** $\sqrt[3]{0.371452\ldots}$ **OR** $\sqrt[3]{\frac{12}{\pi(2\pi+4)}}$ **OR** $\sqrt[3]{\frac{3.81971}{10.2832\ldots}}$ ***(A1)***

$r = 0.719\ (\mathrm{m})$ ***(AG)***

Note: The rounded and unrounded or formulaic answers must be seen for the final ***(A1)*** to be awarded. The use of 3.14 gives an unrounded answer of $r = 0.719039\ldots$.

[2 marks]

f. $0.75 = \frac{\pi \times (0.719)^2 l}{2}$ ***(M1)***

Note: Award ***(M1)*** for substituting 0.719 into their volume formula. Follow through from part (b).

$l = 0.924\ (\mathrm{m})\ \ (0.923599\ldots)$ ***(A1)*****(ft)*****(G2)***

[2 marks]

g. $T = (2\pi + 4) \times 0.719 + \frac{6}{\pi(0.719)^2}$ ***(M1)***

Notes: Award ***(M1)*** for substituting 0.719 in their expression for T. Accept alternative methods, for example substitution of their l and 0.719 into their part (a) (for which the answer is 11.08961024). Follow through from their answer to part (a).

$= 11.1\ (\mathrm{m})\ \ (11.0880\ldots)$ ***(A1)*****(ft)*****(G2)***

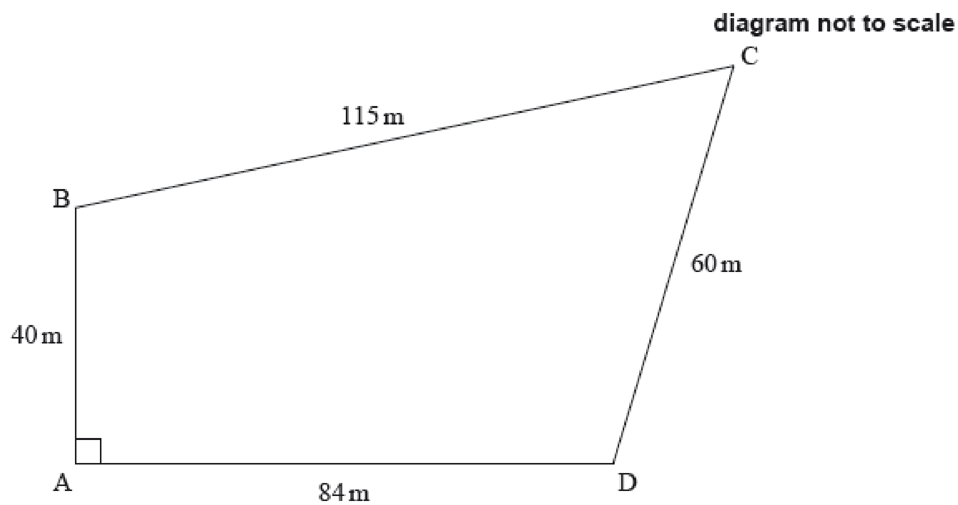
Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]

Abdallah owns a plot of land, near the river Nile, in the form of a quadrilateral ABCD.

The lengths of the sides are $\mathrm{AB} = 40\ \mathrm{m}$, $\mathrm{BC} = 115\ \mathrm{m}$, $\mathrm{CD} = 60\ \mathrm{m}$, $\mathrm{AD} = 84\ \mathrm{m}$ and angle $\mathrm{B\hat{A}D} = 90°$.

This information is shown on the diagram.



The formula that the ancient Egyptians used to estimate the area of a quadrilateral ABCD is

$$\text{area} = \frac{(\mathrm{AB}+\mathrm{CD})(\mathrm{AD}+\mathrm{BC})}{4}.$$

Abdallah uses this formula to estimate the area of his plot of land.

a. Show that $\mathrm{BD} = 93\ \mathrm{m}$ correct to the nearest metre. [2]

b. Calculate angle $\mathrm{B\hat{C}D}$. [3]

c. Find the area of ABCD. [4]

d.i. Calculate Abdallah's estimate for the area. [2]

d.ii. Find the percentage error in Abdallah's estimate. [2]

Markscheme

a. $\mathrm{BD}^2 = 40^2 + 84^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras.

Accept correct substitution into cosine rule.

$\mathrm{BD} = 93.0376\ldots$ ***(A1)***

$= 93$ ***(AG)***

Note: Both the rounded and unrounded value must be seen for the ***(A1)*** to be awarded.

[2 marks]

b. $\cos C = \frac{115^2+60^2-93^2}{2\times115\times60}$ $(93^2 = 115^2 + 60^2 - 2\times115\times60\times\cos C)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into cosine formula, ***(A1)*** for correct substitutions.

$= 53.7^\circ\ (53.6679\ldots^\circ)$ ***(A1)(G2)***

[3 marks]

c. $\frac{1}{2}(40)(84) + \frac{1}{2}(115)(60)\sin(53.6679\ldots)$ ***(M1)(M1)(A1)*(ft)**

Note: Award ***(M1)*** for correct substitution into right-angle triangle area. Award ***(M1)*** for substitution into area of triangle formula and ***(A1)*(ft)** for correct substitution.

$= 4460\ \mathrm{m}^2\ (4459.30\ldots\ \mathrm{m}^2)$ ***(A1)*(ft)*(G3)***

Notes: Follow through from part (b).

[4 marks]

d.i. $\frac{(40+60)(84+115)}{4}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the area formula used by 'Ancient Egyptians'.

$= 4980\ \mathrm{m}^2\ (4975\ \mathrm{m}^2)$ ***(A1)(G2)***

[2 marks]

d.ii. $\left|\frac{4975-4459.30\ldots}{4459.30\ldots}\right| \times 100$ ***(M1)***

Notes: Award ***(M1)*** for correct substitution into percentage error formula.

$= 11.6\ (\%)\ (11.5645\ldots)$ ***(A1)*(ft)*(G2)***

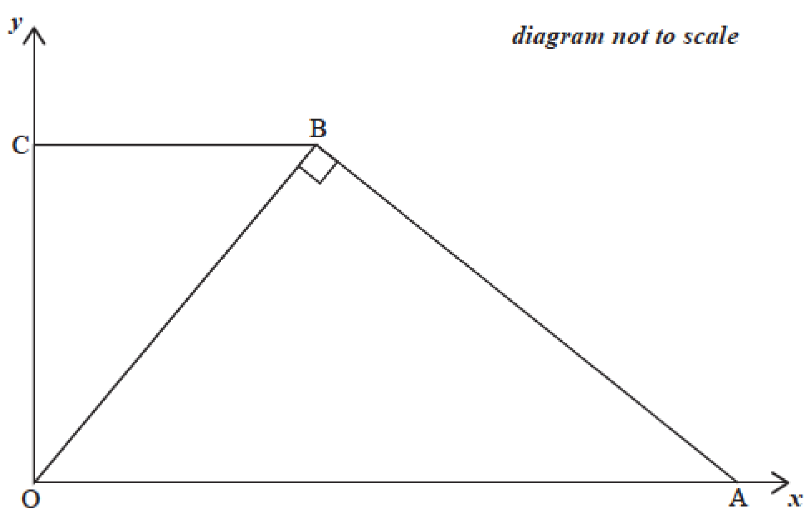
Notes: Follow through from parts (c) and (d)(i).

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d.i. [N/A]
d.ii. [N/A]

The following diagram shows two triangles, OBC and OBA, on a set of axes. Point C lies on the y-axis, and O is the origin.



a. The equation of the line BC is $y = 4$. [1]

Write down the coordinates of point C.

b. The x-coordinate of point B is a. [2]

(i) Write down the coordinates of point B;

(ii) Write down the gradient of the line OB.

c. Point A lies on the x-axis and the line AB is perpendicular to line OB. [4]

(i) Write down the gradient of line AB.

(ii) Show that the equation of the line AB is $4y + ax - a^2 - 16 = 0$.

d. The area of triangle AOB is **three times** the area of triangle OBC. [3]

Find an expression, **in terms of *a***, for

(i) the area of triangle OBC;

(ii) the *x*-coordinate of point A.

e. Calculate the value of a. [2]

Markscheme

a. $(0,\ 4)$ ***(A1)***

Notes: Accept $x = 0,\ y = 4$.

b. (i) $(a,\ 4)$ ***(A1)*(ft)**

Notes: Follow through from part (a).

(ii) $\frac{4}{a}$ ***(A1)*(ft)**

Note: Follow through from part (b)(i).

c. (i) $-\frac{a}{4}$ ***(A1)*(ft)**

Note: Follow through from part (b)(ii).

(ii) $y = -\frac{a}{4}x + c$ ***(M1)***

Note: Award ***(M1)*** for substitution of their gradient from part (c)(i) in the equation.

$4 = -\frac{a}{4} \times a + c$

$c = \frac{1}{4} \times a^2 + 4$

$y = -\frac{a}{4}x + \frac{1}{4}a^2 + 4$ ***(A1)***

OR

$y - 4 = -\frac{a}{4}(x - a)$ ***(M1)***

Note: Award ***(M1)*** for substitution of their gradient from part (c)(i) in the equation.

$y = -\frac{ax}{4} + \frac{a^2}{4} + 4$ ***(A1)***

$4y = -ax + a^2 + 16$

$4y + ax - a^2 - 16 = 0$ ***(AG)***

Note: Both the simplified and the not simplified equations must be seen for the final ***(A1)*** to be awarded.

d. (i) $2a$ ***(A1)***

(ii) $\frac{4x}{2} = 3 \times 2a$ ***(M1)***

Note: Award ***(M1)*** for correct equation.

$x = 3a$ ***(A1)*(ft)**

Note: Follow through from part (d)(i).

OR

$0 - 4 = -\frac{a}{4}(x - a)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their gradient and the coordinates of their point into the equation of a line.

$\frac{16}{a} = x - a$

$x = a + \frac{16}{a}$ ***(A1)*(ft)**

Note: Follow through from parts (b)(i) and (c)(i).

OR

$4 \times 0 + ax - a^2 - 16 = 0$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of the coordinates of $\mathrm{A}(x,\ 0)$ into the equation of line AB.

$ax - a^2 - 16 = 0$

$x = a + \frac{16}{a}$ **OR** $x = \frac{(a^2+16)}{a}$ ***(A1)(G1)***

e. $4(0) + a(3a) - a^2 - 16 = 0$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their $3a$ from part (d)(ii) into the equation of line AB.

OR

$\frac{1}{2}\left(a + \frac{16}{a}\right) \times 4 = 3\left(\frac{4a}{2}\right)$ ***(M1)***

Note: Award ***(M1)*** for area of triangle AOB (with their substituted $a + \frac{16}{a}$ and 4) equated to three times their area of triangle AOB.

$a = 2.83$ $\left(2.82842...,\ 2\sqrt{2},\ \sqrt{8}\right)$ ***(A1)*(ft)*(G1)***

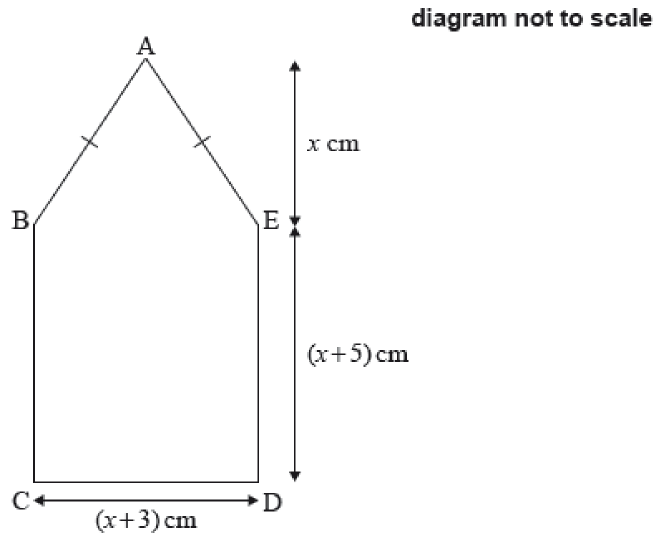
Note: Follow through from parts (d)(i) and (d)(ii).

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

The base of an electric iron can be modelled as a pentagon ABCDE, where:

BCDE is a rectangle with sides of length $(x+3)$ cm and $(x+5)$ cm;
ABE is an isosceles triangle, with $\mathrm{AB} = \mathrm{AE}$ and a height of x cm;
the area of ABCDE is $222\ \mathrm{cm}^2$.



Insulation tape is wrapped around the perimeter of the base of the iron, ABCDE.

F is the point on AB such that $\mathrm{BF} = 8\ \mathrm{cm}$. A heating element in the iron runs in a straight line, from C to F.

a.i. Write down an **equation** for the area of ABCDE using the above information. [2]

a.ii. Show that the equation in part (a)(i) simplifies to $3x^2 + 19x - 414 = 0$. [2]

b. Find the length of CD. [2]

c. Show that angle $\mathrm{B\hat{A}E} = 67.4^\circ$, correct to one decimal place. [3]

d. Find the length of the perimeter of ABCDE. [3]

e. Calculate the length of CF. [4]

Markscheme

a.i. $222 = \frac{1}{2}x(x+3) + (x+3)(x+5)$ ***(M1)(M1)(A1)***

Note: Award ***(M1)*** for correct area of triangle, ***(M1)*** for correct area of rectangle, ***(A1)*** for equating the sum to 222.

OR

$222 = (x+3)(2x+5) - 2\left(\frac{1}{4}\right)x(x+3)$ ***(M1)(M1)(A1)***

Note: Award ***(M1)*** for area of bounding rectangle, ***(M1)*** for area of triangle, ***(A1)*** for equating the difference to 222.

[2 marks]

a.ii. $222 = \frac{1}{2}x^2 + \frac{3}{2}x + x^2 + 3x + 5x + 15$ ***(M1)***

Note: Award ***(M1)*** for complete expansion of the brackets, leading to the final answer, with no incorrect working seen. The final answer must be seen to award ***(M1)***.

$3x^2 + 19x - 414 = 0$ ***(AG)***

[2 marks]

b. $x = 9 \left(\text{and } x = -\frac{46}{3}\right)$ ***(A1)***

$\text{CD} = 12\ (\text{cm})$ ***(A1)(G2)***

[2 marks]

c. $\frac{1}{2}(\text{their } x + 3) = 6$ ***(A1)*(ft)**

Note: Follow through from part (b).

$\tan\left(\frac{\text{B}\hat{\text{A}}\text{E}}{2}\right) = \frac{6}{9}$ ***(M1)***

Note: Award ***(M1)*** for their correct substitutions in tangent ratio.

$\text{B}\hat{\text{A}}\text{E} = 67.3801\ldots^\circ$ ***(A1)***

$= 67.4^\circ$ ***(AG)***

Note: Do not award the final ***(A1)*** unless both the correct unrounded and rounded answers are seen.

OR

$\frac{1}{2}(\text{their } x + 3) = 6$ ***(A1)*(ft)**

$\tan(\text{A}\hat{\text{B}}\text{E}) = \frac{9}{6}$ ***(M1)***

Note: Award ***(M1)*** for their correct substitutions in tangent ratio.

$\text{B}\hat{\text{A}}\text{E} = 180^\circ - 2(\text{A}\hat{\text{B}}\text{E})$

$\text{B}\hat{\text{A}}\text{E} = 67.3801\ldots^\circ$ ***(A1)***

$= 67.4^\circ$ ***(AG)***

Note: Do not award the final ***(A1)*** unless both the correct unrounded and rounded answers are seen.

[3 marks]

d. $2\sqrt{9^2+6^2}+12+2(14)$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras. Award ***(M1)*** for the addition of 5 sides of the pentagon, consistent with their x.

$61.6\ (\mathrm{cm})\ \ (61.6333\ldots\ (\mathrm{cm}))$ ***(A1)*(ft)*(G3)***

Note: Follow through from part (b).

[3 marks]

e. $\mathrm{F\hat{B}C}=90+\left(\frac{180-67.4}{2}\right)\ (=146.3^\circ)$ ***(M1)***

OR

$180-\frac{67.4}{2}$ ***(M1)***

$\mathrm{CF}^2=8^2+14^2-2(8)(14)\cos(146.3^\circ)$ ***(M1)(A1)*(ft)**

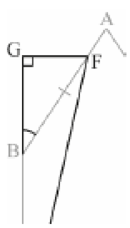
Note: Award ***(M1)*** for substituted cosine rule formula and ***(A1)*** for correct substitutions. Follow through from part (b).

$\mathrm{CF}=21.1\ (\mathrm{cm})\ (21.1271\ldots)$ ***(A1)*(ft)*(G3)***

OR

$\mathrm{G\hat{B}F}=\frac{67.4}{2}=33.7^\circ$ ***(A1)***

Note: Award ***(A1)*** for angle $\mathrm{G\hat{B}F}=33.7^\circ$, where G is the point such that CG is a projection/extension of CB and triangles BGF and CGF are right-angled triangles. The candidate may use another variable.



$\mathrm{GF}=8\sin 33.7^\circ=4.4387\ldots$ **AND** $\mathrm{BG}=8\cos 33.7^\circ=6.6556\ldots$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into trig formulas to find both GF and BG.

$\mathrm{CF}^2=(14+6.6556\ldots)^2+(4.4387\ldots)^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras formula to find CF.

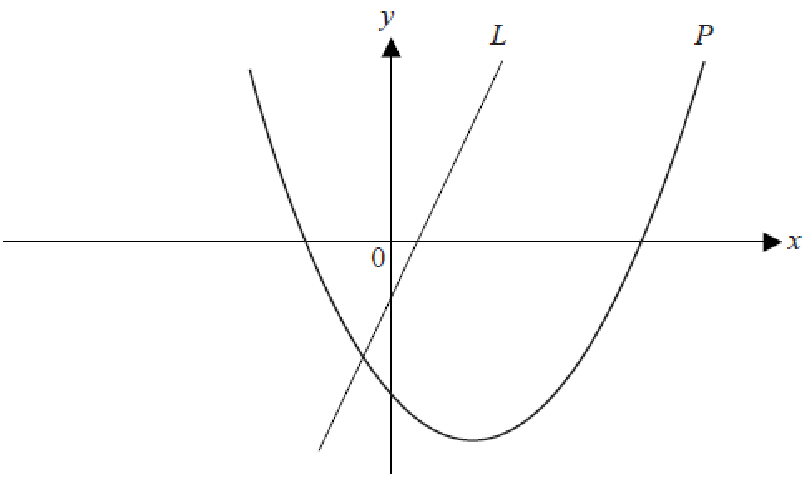
$\mathrm{CF}=21.1\ (\mathrm{cm})\ (21.1271\ldots)$ ***(A1)*(ft)*(G3)***

[4 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

The diagram below shows the graph of a line L passing through (1, 1) and (2 , 3) and the graph P of the function $f(x) = x^2 - 3x - 4$



a. Find the gradient of the line *L*. [2]

b. Differentiate $f(x)$. [2]

c. Find the coordinates of the point where the tangent to *P* is parallel to the line *L*. [3]

d. Find the coordinates of the point where the tangent to *P* is perpendicular to the line *L*. [4]

e. Find [3]

(i) the gradient of the tangent to *P* at the point with coordinates (2, – 6).

(ii) the equation of the tangent to *P* at this point.

f. State the equation of the axis of symmetry of *P*. [1]

g. Find the coordinates of the vertex of *P* and state the gradient of the curve at this point. [3]

Markscheme

a. *for attempt at substituted* $\frac{y distance}{x distance}$ ***(M1)***

gradient = 2 ***(A1)(G2)***

[2 marks]

b. $2x - 3$ ***(A1)(A1)***

(A1) *for* $2x$, ***(A1)*** *for* -3

[2 marks]

c. *for their* $2x - 3 =$ *their gradient and attempt to solve* ***(M1)***

$x = 2.5$ ***(A1)*(ft)**

$y = -5.25$ *(**(ft)** from their x value)* ***(A1)*(ft)*(G2)***

[3 marks]

d. *for seeing* $\frac{-1}{their(a)}$ ***(M1)***

solving $2x - 3 = -\frac{1}{2}$ *(or their value)* ***(M1)***

x = 1.25 ***(A1)*(ft)*(G1)***

y = – 6.1875 ***(A1)*(ft)*(G1)***

[4 marks]

e. (i) $2 \times 2 - 3 = 1$ *(**(ft)** from (b))* ***(A1)*(ft)*(G1)***

(ii) $y = mx + c$ *or equivalent method to find* $c \Rightarrow -6 = 2 + c$ ***(M1)***

$y = x - 8$ ***(A1)*(ft)*(G2)***

[3 marks]

f. $x = 1.5$ ***(A1)***

[1 mark]

g. *for substituting their answer to part (f) into the equation of the parabola* (1.5, –6.25) *accept x* = 1.5, *y* = –6.25 ***(M1)(A1)*(ft)*(G2)***

gradient is zero *(accept* $\frac{dy}{dx} = 0$*)* ***(A1)***

[3 marks]

Examiners report

a. Parts (a) and (b) were very well done. After that, only the stronger candidates were able to cope. The equation of the tangent at the point with coordinates (2, 6) was badly done but some candidates managed to find the equation of the tangent line from their GDC. The equation of the axis of symmetry was reasonably well done although many just wrote down 1.5 instead of *x* = 1.5.

Some forgot to write down that the gradient at the vertex was 0.

b. Parts (a) and (b) were very well done. After that, only the stronger candidates were able to cope. The equation of the tangent at the point with coordinates (2, 6) was badly done but some candidates managed to find the equation of the tangent line from their GDC. The equation of the axis of symmetry was reasonably well done although many just wrote down 1.5 instead of *x* = 1.5.

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Some forgot to write down that the gradient at the vertex was 0.

e. Parts (a) and (b) were very well done. After that, only the stronger candidates were able to cope. The equation of the tangent at the point with coordinates (2, 6) was badly done but some candidates managed to find the equation of the tangent line from their GDC. The equation of the axis of symmetry was reasonably well done although many just wrote down 1.5 instead of $x = 1.5$.

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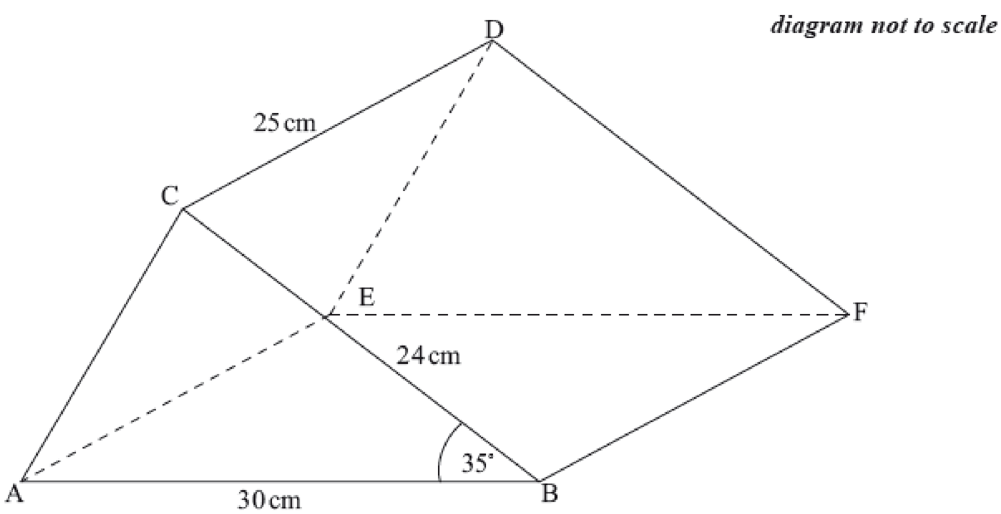
Some forgot to write down that the gradient at the vertex was 0.

A manufacturer has a contract to make 2600 solid blocks of wood. Each block is in the shape of a right triangular prism, ABCDEF, as shown in the diagram.

$\mathrm{AB} = 30\text{ cm},\ \mathrm{BC} = 24\text{ cm},\ \mathrm{CD} = 25\text{ cm}$ and angle $\mathrm{A\hat{B}C} = 35^\circ$.



a. Calculate the length of AC. [3]

b. Calculate the area of triangle ABC. [3]

c. Assuming that no wood is wasted, show that the volume of wood required to make all 2600 blocks is $13\,400\,000\ \text{cm}^3$, correct to three significant figures. [2]

d. Write $13\,400\,000$ in the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$. [2]

e. Show that the total surface area of one block is $2190\ \text{cm}^2$, correct to three significant figures. [3]

f. The blocks are to be painted. One litre of paint will cover $22\ \text{m}^2$. [3]

Calculate the number of litres required to paint all 2600 blocks.

Markscheme

a. $\text{AC}^2 = 30^2 + 24^2 - 2 \times 30 \times 24 \times \cos 35^\circ$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted cosine rule formula,

(A1) for correct substitutions.

$\text{AC} = 17.2\ \text{cm} \quad (17.2168\ldots)$ ***(A1)(G2)***

Notes: Use of radians gives $52.7002\ldots$ Award ***(M1)(A1)(A0)***.

No marks awarded in this part of the question where candidates assume that angle $\text{ACB} = 90^\circ$.

[3 marks]

b. ***Units are required in part (b).***

Area of triangle $\text{ABC} = \frac{1}{2} \times 24 \times 30 \times \sin 35^\circ$ ***(M1)(A1)***

Notes: Award ***(M1)*** for substitution into area formula, ***(A1)*** for correct substitutions.

Special Case: Where a candidate has assumed that angle $\text{ACB} = 90^\circ$ in part (a), award ***(M1)(A1)*** for a correct alternative substituted formula for the area of the triangle $\left(ie\ \frac{1}{2} \times \text{base} \times \text{height}\right)$.

$= 206\ \text{cm}^2 \quad (206.487\ldots\text{cm}^2)$ ***(A1)(G2)***

Notes: Use of radians gives negative answer, $-154.145\ldots$ Award (***M1)(A1)(A0)***.

Special Case: Award ***(A1)*(ft)** where the candidate has arrived at an area which is correct to the standard rounding rules from their lengths (units required).

[3 marks]

c. $206.487\ldots \times 25 \times 2600$ ***(M1)***

Note: Award ***(M1)*** for multiplication of their answer to part (b) by 25 and 2600.

$13\,421\,688.61$ ***(A1)***

Note: Accept unrounded answer of $13\,390\,000$ for use of 206.

$13\,400\,000$ ***(AG)***

Note: The final ***(A1)*** cannot be awarded unless both the unrounded and rounded answers are seen.

[2 marks]

d. 1.34×10^7 ***(A2)***

Notes: Award ***(A2)*** for the correct answer.

Award ***(A1)(A0)*** for 1.34 and an incorrect index value.

Award ***(A0)(A0)*** for any other combination (including answers such as 13.4×10^6).

[2 marks]

e. $2 \times 206.487\ldots + 24 \times 25 + 30 \times 25 + 17.2168\ldots \times 25$ ***(M1)(M1)***

Note: Award ***(M1)*** for multiplication of their answer to part (b) by 2 for area of two triangular ends, ***(M1)*** for three correct rectangle areas using 24, 30 and their 17.2.

$2193.26\ldots$ ***(A1)***

Note: Accept 2192 for use of 3 sf answers.

2190 ***(AG)***

Note: The final ***(A1)*** cannot be awarded unless both the unrounded and rounded answers are seen.

[3 marks]

f. $\frac{2190 \times 2600}{22 \times 10\,000}$ ***(M1)(M1)***

Notes: Award ***(M1)*** for multiplication by 2600 and division by 22, ***(M1)*** for division by $10\,000$.

The use of 22 may be implied *ie* division by 2200 would be acceptable.

25.9 litres $(25.8818\ldots)$ ***(A1)(G2)***

Note: Accept 26.

[3 marks]

Examiners report

a. Some candidates assumed that triangle ACB was a right angled triangle with angle $\mathrm{ACB} = 90^\circ$. Such candidates earned no marks for part (a) but were able to recover most of the marks in the remainder of the question. For those candidates who correctly used the cosine rule for part (a), most achieved all 3 marks for this part and used a correct formula for the area of the triangle in part (b) to obtain at least 2 marks for this part. The final mark was not awarded, however, if no units or the incorrect units were given. Parts (c) and (e) were generally well done with many candidates showing the unrounded answer before the required answer. Part (f) proved to be quite problematic for many candidates. Whilst many were able to earn a method mark for $\frac{2190\times 2600}{22}$, a significant number of these candidates were unable to convert the units correctly and very few correct answers were seen. Indeed, the most popular answer seemed to be 2590 litres.

b. Some candidates assumed that triangle ACB was a right angled triangle with angle $\mathrm{ACB} = 90^\circ$. Such candidates earned no marks for part (a) but were able to recover most of the marks in the remainder of the question. For those candidates who correctly used the cosine rule for part (a), most achieved all 3 marks for this part and used a correct formula for the area of the triangle in part (b) to obtain at least 2 marks for this part. The final mark was not awarded, however, if no units or the incorrect units were given. Parts (c) and (e) were generally well done with many candidates showing the unrounded answer before the required answer. Part (f) proved to be quite problematic for many candidates. Whilst many were able to earn a method mark for $\frac{2190\times 2600}{22}$, a significant number of these candidates were unable to convert the units correctly and very few correct answers were seen. Indeed, the most popular answer seemed to be 2590 litres.

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Consider the curve $y = x^3 + \frac{3}{2}x^2 - 6x - 2$.

a. (i) Write down the value of y when x is 2. [3]

(ii) Write down the coordinates of the point where the curve intercepts the y-axis.

b. Sketch the curve for $-4 \leqslant x \leqslant 3$ and $-10 \leqslant y \leqslant 10$. Indicate clearly the information found in (a). [4]

c. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

d. Let L_1 be the tangent to the curve at $x = 2$. [8]

Let L_2 be a tangent to the curve, parallel to L_1.

(i) Show that the gradient of L_1 is 12.

(ii) Find the x-coordinate of the point at which L_2 and the curve meet.

(iii) Sketch and label L_1 and L_2 on the diagram drawn in (b).

e. It is known that $\frac{\mathrm{d}y}{\mathrm{d}x} > 0$ for $x < -2$ and $x > b$ where b is positive. [5]

(i) Using your graphic display calculator, or otherwise, find the value of b.

(ii) Describe the behaviour of the curve in the interval $-2 < x < b$.

(iii) Write down the equation of the tangent to the curve at $x = -2$.

Markscheme

a. (i) $y = 0$ ***(A1)***

(ii) $(0, -2)$ ***(A1)(A1)***

Note: Award ***(A1)(A0)*** if brackets missing.

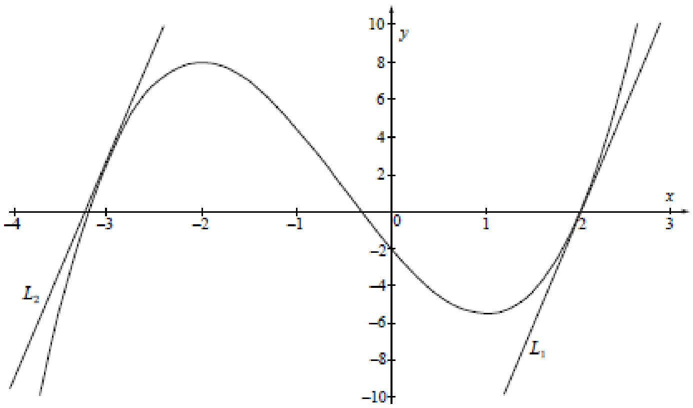
OR

$x = 0, y = -2$ ***(A1)(A1)***

Note: If coordinates reversed award ***(A0)(A1)*****(ft)**. Two coordinates must be given.

[3 marks]

b.



(A4)

Notes: ***(A1)*** for appropriate window. Some indication of scale on the x-axis must be present (for example ticks). Labels not required. ***(A1)*** for smooth curve and shape, ***(A1)*** for maximum and minimum in approximately correct position, ***(A1)*** for x and y intercepts found in (a) in approximately correct position.

[4 marks]

c. $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 3x - 6$ ***(A1)(A1)(A1)***

Note: ***(A1)*** for each correct term. Award ***(A1)(A1)(A0)*** at most if any other term is present.

[3 marks]

d. (i) $3 \times 4 + 3 \times 2 - 6 = 12$ ***(M1)(A1)(AG)***

Note: ***(M1)*** for using the derivative and substituting $x = 2$. ***(A1)*** for correct (and clear) substitution. The 12 must be seen.

(ii) Gradient of L_2 is 12 (can be implied) ***(A1)***

$3x^2 + 3x - 6 = 12$ ***(M1)***

$x = -3$ ***(A1)(G2)***

Note: ***(M1)*** for equating the derivative to 12 or showing a sketch of the derivative together with a line at $y = 12$ or a table of values showing the 12 in the derivative column.

(iii) ***(A1)*** for L_1 correctly drawn at approx the correct point ***(A1)***

(A1) for L_2 correctly drawn at approx the correct point ***(A1)***

(A1) for 2 parallel lines ***(A1)***

Note: If lines are not labelled award at most ***(A1)(A1)(A0)***. Do not accept 2 horizontal or 2 vertical parallel lines.

[8 marks]

e. (i) $b = 1$ ***(G2)***

(ii) The curve is decreasing. ***(A1)***

Note: Accept any valid description.

(iii) $y = 8$ ***(A1)(A1)(G2)***

Note: ***(A1)*** for "$y =$ a constant", ***(A1)*** for 8.

[5 marks]

Examiners report

a. Many candidates managed to gain good marks in this question as they were able to answer the first three parts of the question. Good sketches were drawn with the required information shown on them. Very few candidates did not recognise the notation $\frac{\mathrm{d}y}{\mathrm{d}x}$ but they showed that they knew how to differentiate as in (d)(i) they found the derivative to show that the gradient of L_1 was 12. Candidates found it difficult to find the other x for which the derivative was 12. However, some could draw both tangents without having found this value of x . In general, tangents were not well drawn. The last part question did act as a discriminating question. However, those candidates that had the function drawn either in their GDC or on paper recognised that at $x = -2$ there was a maximum and so wrote down the correct equation of the tangent at that point.

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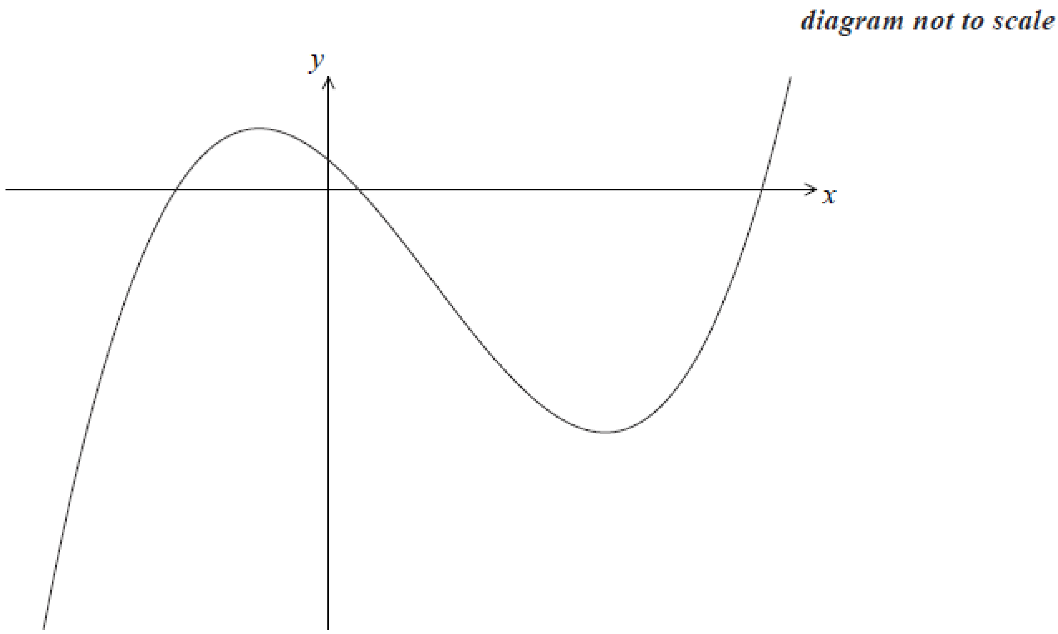
The diagram shows a sketch of the function $f(x) = 4x^3 - 9x^2 - 12x + 3$.



a. Write down the values of *x* where the graph of *f* (*x*) intersects the *x*-axis. [3]

b. Write down *f* ′(*x*). [3]

c. Find the value of the local maximum of *y* = *f* (*x*). [4]

d. Let P be the point where the graph of *f* (*x*) intersects the *y* axis. [1]

Write down the coordinates of P.

e. Let P be the point where the graph of *f* (*x*) intersects the *y* axis. [2]

Find the gradient of the curve at P.

f. The line, *L*, is the tangent to the graph of *f* (*x*) at P. [2]

Find the equation of *L* in the form *y* = *mx* + *c*.

g. There is a second point, Q, on the curve at which the tangent to *f* (*x*) is parallel to *L*. [1]

Write down the gradient of the tangent at Q.

h. There is a second point, Q, on the curve at which the tangent to *f* (*x*) is parallel to *L*. [3]

Calculate the *x*-coordinate of Q.

Markscheme

a. -1.10, 0.218, 3.13 ***(A1)(A1)(A1)***

[3 marks]

b. $f'(x) = 12x^2 - 18x - 12$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for each correct term and award maximum of ***(A1)(A1)*** if other terms seen.

[3 marks]

c. $f'(x) = 0$ ***(M1)***

$x = -0.5, 2$

$x = -0.5$ ***(A1)***

Note: If $x = -0.5$ not stated, can be inferred from working below.

$y = 4(-0.5)^3 - 9(-0.5)^2 - 12(-0.5) + 3$ ***(M1)***
$y = 6.25$ ***(A1)(G3)***

Note: Award ***(M1)*** for their value of x substituted into $f(x)$.

Award ***(M1)(G2)*** if sketch shown as method. If coordinate pair given then award ***(M1)(A1)(M1)(A0)***. If coordinate pair given with no working award ***(G2)***.

[4 marks]

d. $(0, 3)$ ***(A1)***

Note: Accept $x = 0$, $y = 3$.

[1 mark]

e. $f'(0) = -12$ ***(M1)(A1)*****(ft)*****(G2)***

Note: Award ***(M1)*** for substituting $x = 0$ into their derivative.

[2 marks]

f. Tangent: $y = -12x + 3$ ***(A1)*****(ft)*****(A1)(G2)***

Note: Award ***(A1)*****(ft)** for their gradient, ***(A1)*** for intercept = 3.

Award ***(A1)(A0)*** if $y =$ not seen.

[2 marks]

g. -12 ***(A1)*****(ft)**

Note: Follow through from their part (e).

[1 mark]

h. $12x^2 - 18x - 12 = -12$ ***(M1)***

$12x^2 - 18x = 0$ ***(M1)***

$x = 1.5, 0$

At Q $x = 1.5$ ***(A1)*(ft)*(G2)***

Note: Award ***(M1)(G2)*** for $12x^2 - 18x - 12 = -12$ followed by $x = 1.5$.

Follow through from their part (g).

[3 marks]

Examiners report

a. This question was either very well done – by the majority – or very poor and incomplete attempts were seen. This would perhaps indicate a lack of preparation in this area of the syllabus from some centres, though it is recognised that the differential calculus is one of the more problematic topics for the candidature.

It was however disappointing to note the number of candidates who do not use the GDC to good effect; in part (a) for example, the zeros were not found accurately due to "trace" being used; this is not a suitable approach – there is a built-in zero finder which should be used. Much of the question was accessible via a GDC approach, a sketch was given that could have been verified on the GDC; this was lost on many.

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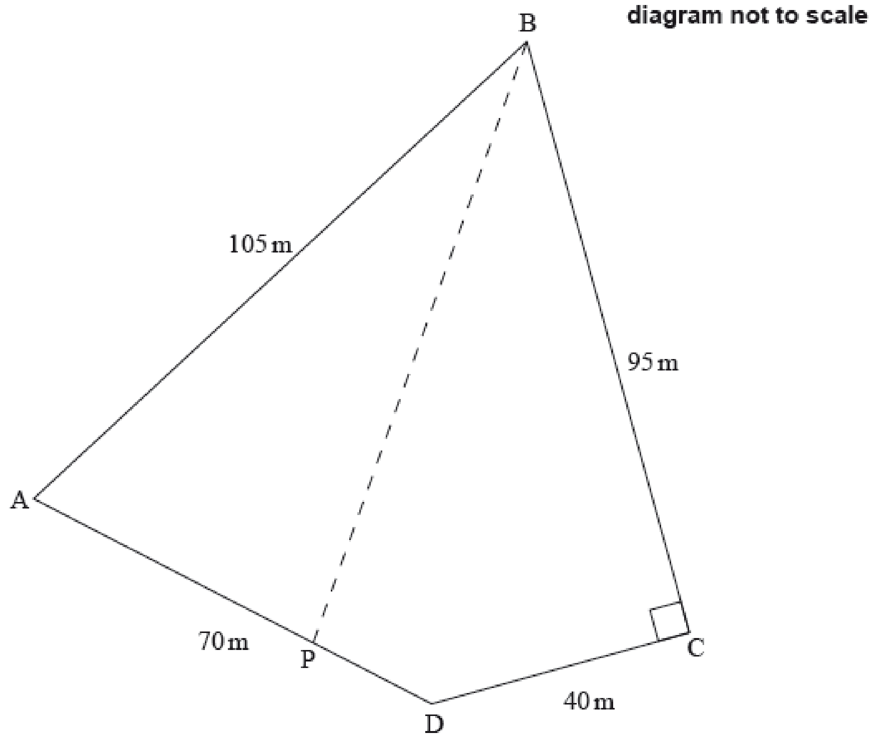
A farmer owns a plot of land in the shape of a quadrilateral ABCD.

$\mathrm{AB} = 105\ \mathrm{m}$, $\mathrm{BC} = 95\ \mathrm{m}$, $\mathrm{CD} = 40\ \mathrm{m}$, $\mathrm{DA} = 70\ \mathrm{m}$ and angle $\mathrm{DCB} = 90^\circ$.



The farmer wants to divide the land into two equal areas. He builds a fence in a straight line from point B to point P on AD, so that the area of PAB is equal to the area of PBCD.

Calculate

a. the length of BD; [2]

b. the size of angle DAB; [3]

c. the area of triangle ABD; [3]

d. the area of quadrilateral ABCD; [2]

e. the length of AP; [3]

f. the length of the fence, BP. [3]

Markscheme

a. $(\mathrm{BD} =)\ \sqrt{95^2 + 40^2}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras' theorem.

$= 103\ (\mathrm{m})\ \left(103.077\ldots,\ 25\sqrt{17}\right)$ ***(A1)(G2)***

[2 marks]

b. $\cos \mathrm{B\hat{A}D} = \frac{105^2+70^2-(103.077\ldots)^2}{2\times105\times70}$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substitution into cosine rule, ***(A1)*****(ft)** for their correct substitutions. Follow through from part (a).

$(\mathrm{B\hat{A}D}) = 68.9° \ (68.8663\ldots)$ ***(A1)*(ft)*(G2)***

Note: If their 103 used, the answer is $68.7995\ldots$

[3 marks]

c. $(\text{Area of ABD} =) \frac{1}{2} \times 105 \times 70 \times \sin(68.8663\ldots)$ ***(M1)(A1)*(ft)**

Notes: Award ***(M1)*** for substitution into the trig form of the area of a triangle formula.

Award ***(A1)*(ft)** for their correct substitutions.

Follow through from part (b).

If 68.8° is used the area $= 3426.28\ldots\ \mathrm{m}^2$.

$= 3430\ \mathrm{m}^2\ (3427.82\ldots)$ ***(A1)*(ft)*(G2)***

[3 marks]

d. $\text{area of ABCD} = \frac{1}{2} \times 40 \times 95 + 3427.82\ldots$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted area of triangle formula **added** to their answer to part (c).

$= 5330\ \mathrm{m}^2\ (5327.83\ldots)$ ***(A1)*(ft)*(G2)***

[2 marks]

e. $\frac{1}{2} \times 105 \times \mathrm{AP} \times \sin(68.8663\ldots) = 0.5 \times 5327.82\ldots$ ***(M1)(M1)***

Notes: Award ***(M1)*** for the correct substitution into triangle formula.

Award ***(M1)*** for equating their triangle area to half their part (d).

$(\mathrm{AP} =)\ 54.4\ (\mathrm{m})\ (54.4000\ldots)$ ***(A1)*(ft)*(G2)***

Notes: Follow through from parts (b) and (d).

[3 marks]

f. $\mathrm{BP}^2 = 105^2 + (54.4000\ldots)^2 - 2 \times 105 \times (54.4000\ldots) \times \cos(68.8663\ldots)$ ***(M1)(A1)*(ft)**

Notes: Award ***(M1)*** for substituted cosine rule formula.

Award ***(A1)*(ft)** for their correct substitutions. Accept the exact fraction $\frac{53}{147}$ in place of $\cos(68.8663\ldots)$.

Follow through from parts (b) and (e).

$(\mathrm{BP} =)\ 99.3\ (\mathrm{m})\ (99.3252\ldots)$ ***(A1)*(ft)*(G2)***

Notes: If 54.4 and $\cos(68.9)$ are used the answer is $99.3567\ldots$

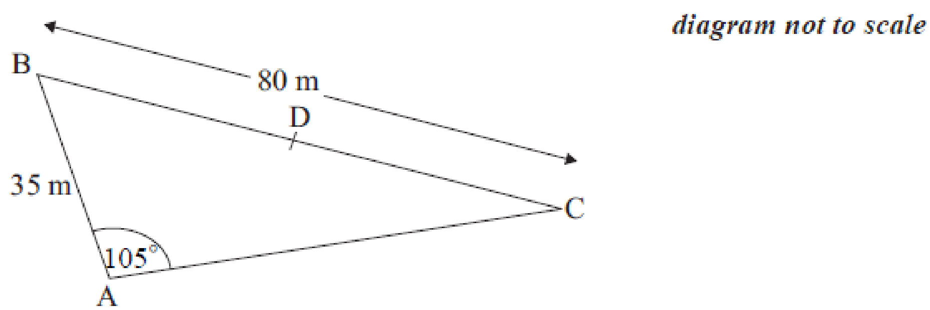
[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

A farmer has a triangular field, ABC, as shown in the diagram.

AB = 35 m, BC = 80 m and BÂC = 105°, and D is the midpoint of BC.



a. Find the size of BĈA. [3]

b. Calculate the length of AD. [5]

c. The farmer wants to build a fence around ABD. [2]

Calculate the total length of the fence.

d. The farmer wants to build a fence around ABD. [2]

The farmer pays 802.50 USD for the fence. Find the cost per metre.

e. Calculate the area of the triangle ABD. [3]

f. A layer of earth 3 cm thick is removed from ABD. Find the volume removed in cubic metres. [3]

Markscheme

a. $\frac{\sin \mathrm{BCA}}{35} = \frac{\sin 105^\circ}{80}$ ***(M1)(A1)***

Note: Award ***(M1)*** for correct substituted formula, ***(A1)*** for correct substitutions.

$\mathrm{B\hat{C}A} = 25.0°$ ***(A1)(G2)***

[3 marks]

b. ***Note: Unit penalty (UP) applies in parts (b)(c) and (e)***

Length BD = 40 m ***(A1)***

Angle ABC = 180° – 105° – 25° = 50° ***(A1)*****(ft)**

Note: (ft) from their answer to (a).

$AD^2 = 35^2 + 40^2 - (2 \times 35 \times 40 \times \cos 50°)$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for correct substituted formula, ***(A1)*****(ft)** for correct substitutions.

(UP) AD = 32.0 m ***(A1)*****(ft)*****(G3)***

Notes: If 80 is used for BD award at most ***(A0)(A1)*****(ft)*****(M1)(A1)*****(ft)*****(A1)*****(ft)** for an answer of 63.4 m.

If the angle ABC is incorrectly calculated **in this part** award at most ***(A1)(A0)(M1)(A1)*****(ft)*****(A1)*****(ft)**.

If angle BCA is used award at most ***(A1)(A0)(M1)(A0)(A0)***.

[5 marks]

c. ***Note: Unit penalty (UP) applies in parts (b)(c) and (e)***

length of fence = 35 + 40 + 32 ***(M1)***

(UP) = 107 m ***(A1)*****(ft)*****(G2)***

Note: ***(M1)*** for adding 35 + 40 + their (b).

[2 marks]

d. cost per metre $= \frac{802.50}{107}$ ***(M1)***

Note: Award ***(M1)*** for dividing 802.50 by their (c).

cost per metre = 7.50 USD (7.5 USD) (USD not required) ***(A1)*****(ft)*****(G2)***

[2 marks]

e. ***Note: Unit penalty (UP) applies in parts (b)(c) and (e)***

Area of ABD $= \frac{1}{2} \times 35 \times 40 \times \sin 50°$ ***(M1)***

= 536.2311102 ***(A1)*****(ft)**

(UP) = 536 m^2 ***(A1)*****(ft)*****(G2)***

Note: Award ***(M1)*** for correct substituted formula, ***(A1)*****(ft)** for correct substitution, **(ft)** from their value of BD and their angle ABC in (b).

[3 marks]

f. Volume = 0.03 × 536 ***(A1)(M1)***

= 16.08

= 16.1 ***(A1)*(ft)*(G2)***

Note: Award ***(A1)*** for 0.03, ***(M1)*** for correct formula. **(ft)** from their (e).

If 3 is used award at most ***(A0)(M1)(A0)***.

[3 marks]

Examiners report

a. This was a simple application of non-right angled trigonometry and most candidates answered it well. Some candidates lost marks in both parts due to the incorrect setting of the calculators. Those that did not score well overall primarily used Pythagoras.

b. This was a simple application of non-right angled trigonometry and most candidates answered it well. Some candidates lost marks in both parts due to the incorrect setting of the calculators. Those that did not score well overall primarily used Pythagoras.

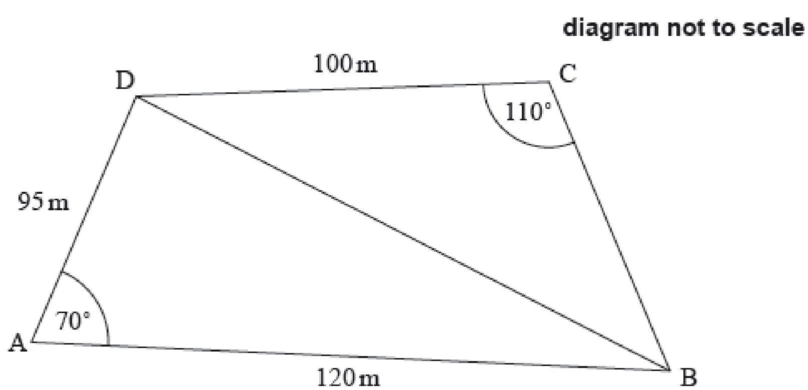
c. Most candidates scored full marks, many by follow through from an incorrect part (b). The main error was using the value for BC and not BD.

d. Most candidates scored full marks, many by follow through from an incorrect part (b). The main error was using the value for BC and not BD.

e. Done well; again some candidates used the right-angled formula.

f. This part was poorly done; many candidates unable to convert 3 cm to 0.03 m. A significant number used the wrong formula, multiplying their answer by 1/3.

The quadrilateral ABCD represents a park, where $\text{AB} = 120\text{ m}$, $\text{AD} = 95\text{ m}$ and $\text{DC} = 100\text{ m}$. Angle DAB is 70° and angle DCB is 110°. This information is shown in the following diagram.



A straight path through the park joins the points B and D.

A new path, CE, is to be built such that E is the point on BD closest to C.

The section of the park represented by triangle DCE will be used for a charity race. A track will be marked along the sides of this section.

a. Find the length of the path BD. [3]

b. Show that angle DBC is 48.7°, correct to three significant figures. [3]

c. Find the area of the park. [4]

d. Find the length of the path CE. [2]

e. Calculate the total length of the track. [3]

Markscheme

a. $(\mathrm{BD}^2 =)\ 95^2 + 120^2 - 2 \times 95 \times 120 \times \cos 70^\circ$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted cosine rule, ***(A1)*** for correct substitution.

$(\mathrm{BD} =)\ 125\ (\mathrm{m})\ \ (125.007\ldots\ (\mathrm{m}))$ ***(A1)(G2)***

[3 marks]

b. $\frac{\sin \mathrm{DBC}}{100} = \frac{\sin 110^\circ}{125.007\ldots}$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substituted sine rule, ***(A1)*****(ft)** for correct substitution.

Follow through from their answer to part (a).

$(\mathrm{DBC} =)\ 48.7384\ldots^\circ$ ***(A1)*****(ft)**

$(\mathrm{DBC} =)\ 48.7^\circ$ ***(AG)***

Notes: Award the final ***(A1)(ft)*** only if both their unrounded answer and 48.7° is seen. Follow through from their answer to part (a), only if their unrounded answer rounds to 48.7°.

[3 marks]

c. $\frac{1}{2} \times 125.007\ldots \times 100 \times \sin 21.3^\circ + \frac{1}{2} \times 95 \times 120 \times \sin 70^\circ$ ***(A1)(M1)(M1)***

Note: Award ***(A1)*** for 21.3° (21.2615…) seen, ***(M1)*** for substitution into (at least) one area of triangle formula in the form $\frac{1}{2}ab \sin c$, ***(M1)*** for **their** correct substitutions and adding the two areas.

$7630\ \mathrm{m}^2\ (7626.70\ldots \mathrm{m}^2)$ ***(A1)*****(ft)*****(G3)***

Notes: Follow through from their answers to part (a). Accept 7620 m^2 $(7622.79\ldots\text{m}^2)$ from use of 48.7384…

[4 marks]

d. $(\text{CE} =)\ 100 \times \sin 21.3^\circ$ ***(M1)***

$(\text{CE} =)\ 36.3\ (\text{m})\ \ (36.3251\ldots\ (\text{m}))$ ***(A1)*(ft)*(G2)***

Note: Follow through from their angle 21.3° in part (c). Award ***(M0)(A0)*** for halving 110° and/or assuming E is the midpoint of BD in any method seen.

OR

$\text{area of BCD} = \frac{1}{2}\text{BD} \times \text{CE}$ ***(M1)***

$(\text{CE} =)\ 36.3\ (\text{m})\ \ (36.3251\ldots\ (\text{m}))$ ***(A1)*(ft)*(G2)***

Note: Follow through from parts (a) and (c). Award ***(M0)(A0)*** for halving 110° and/or assuming E is the midpoint of BD in any method seen.

[2 marks]

e. $\sqrt{100^2 - 36.3251\ldots^2} + 100 + 36.3251\ldots$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct use of Pythagoras to find DE (or correct trigonometric equation, $100 \times \cos 21.3$, to find DE), ***(M1)*** for the sum of 100, their DE and their CE.

$229\ (\text{m})\ \ (229.494\ldots\ (\text{m}))$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (d). Use of 3 sf values gives an answer of $230\ (\text{m})\ \ (229.5\ (\text{m}))$.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

A boat race takes place around a triangular course, ABC, with $\text{AB} = 700\text{ m}$, $\text{BC} = 900\text{ m}$ and angle $\text{ABC} = 110^\circ$. The race starts and finishes at point A.

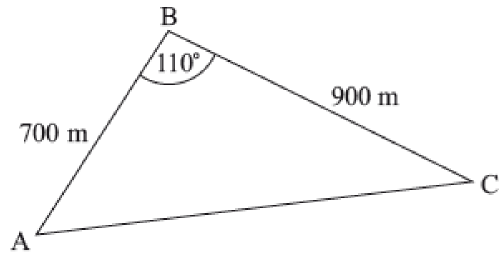


diagram not to scale

a. Calculate the total length of the course. [4]

b. It is estimated that the fastest boat in the race can travel at an average speed of $1.5\ \mathrm{m\,s^{-1}}$. [3]

Calculate an estimate of the winning time of the race. Give your answer to the nearest minute.

c. It is estimated that the fastest boat in the race can travel at an average speed of $1.5\ \mathrm{m\,s^{-1}}$. [3]

Find the size of angle ACB.

d. To comply with safety regulations, the area inside the triangular course must be kept clear of other boats, and the shortest distance from B to AC must be greater than 375 metres. [3]

Calculate the area that must be kept clear of boats.

e. To comply with safety regulations, the area inside the triangular course must be kept clear of other boats, and the shortest distance from B to AC must be greater than 375 metres. [3]

Determine, giving a reason, whether the course complies with the safety regulations.

f. The race is filmed from a helicopter, H, which is flying vertically above point A. [2]

The angle of elevation of H from B is 15°.

Calculate the vertical height, AH, of the helicopter above A.

g. The race is filmed from a helicopter, H, which is flying vertically above point A. [3]

The angle of elevation of H from B is 15°.

Calculate the maximum possible distance from the helicopter to a boat on the course.

Markscheme

a. $\mathrm{AC}^2 = 700^2 + 900^2 - 2 \times 700 \times 900 \times \cos 110^\circ$ ***(M1)(A1)***

$\mathrm{AC} = 1315.65\ldots$ ***(A1)(G2)***

length of course $= 2920\ (\mathrm{m}) \quad (2915.65\ldots\ \mathrm{m})$ ***(A1)***

Notes: Award ***(M1)*** for substitution into cosine rule formula, ***(A1)*** for correct substitution, ***(A1)*** for correct answer.

Award ***(G3)*** for $2920 \quad (2915.65\ldots)$ seen without working.

The final ***(A1)*** is awarded for adding 900 and 700 to their AC irrespective of working seen.

b. $\frac{2915.65}{1.5}$ ***(M1)***

Note: Award ***(M1)*** for their length of course divided by 1.5.

Follow through from part (a).

$= 1943.76\ldots$ (seconds) ***(A1)*(ft)**

$= 32$ (minutes) ***(A1)*(ft)*(G2)***

Notes: Award the final ***(A1)*** for correct conversion of **their** answer in seconds to minutes, correct to the nearest minute.

Follow through from part (a).

c. $\frac{700}{\sin \mathrm{ACB}} = \frac{1315.65\ldots}{\sin 110^\circ}$ ***(M1)(A1)*(ft)**

OR

$\cos \mathrm{ACB} = \frac{900^2 + 1315.65\ldots^2 - 700^2}{2 \times 900 \times 1315.65\ldots}$ ***(M1)(A1)*(ft)**

$\mathrm{ACB} = 30.0^\circ \quad (29.9979\ldots^\circ)$ ***(A1)*(ft)*(G2)***

Notes: Award ***(M1)*** for substitution into sine rule or cosine rule formula, ***(A1)*** for their correct substitution, ***(A1)*** for correct answer.

Accept 29.9° for sine rule and 29.8° for cosine rule from use of correct three significant figure values. Follow through from their answer to (a).

d. $\frac{1}{2} \times 700 \times 900 \times \sin 110^\circ$ ***(M1)(A1)***

Note: Accept $\frac{1}{2} \times \text{their AC} \times 900 \times \sin(\text{their ACB})$. Follow through from parts (a) and (c).

$= 296000 \text{ m}^2 \quad (296003 \text{ m}^2)$ ***(A1)(G2)***

Notes: Award ***(M1)*** for substitution into area of triangle formula, ***(A1)*** for correct substitution, ***(A1)*** for correct answer.

Award ***(G1)*** if 296000 is seen without units or working.

e. $\sin 29.9979\ldots = \frac{\text{distance}}{900}$ ***(M1)***

$(\text{distance} =)\ 450 \text{ (m)} \quad (449.971\ldots)$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (c).

OR

$\frac{1}{2} \times \text{distance} \times 1315.65\ldots = 296003$ ***(M1)***

$(\text{distance} =)\ 450 \text{ (m)} \quad (449.971\ldots)$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (a) and part (d).

450 is greater than 375, thus the course complies with the safety regulations ***(R1)***

Notes: A comparison of their area from (d) and the area resulting from the use of 375 as the perpendicular distance is a valid approach and should be given full credit. Similarly a comparison of angle ACB and $\sin^{-1}\left(\frac{375}{900}\right)$ should be given full credit.

Award ***(R0)*** for correct answer without any working seen. Award ***(R1)*(ft)** for a justified reason consistent with their working.

Do not award ***(M0)(A0)(R1)***.

f. $\tan 15^\circ = \frac{\mathrm{AH}}{700}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into trig formula.

$\mathrm{AH} = 188 \text{ (m)} \quad (187.564\ldots)$ ***(A1)*(ft)*(G2)***

g. $\mathrm{HC}^2 = 187.564\ldots^2 + 1315.65\ldots^2$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into Pythagoras, ***(A1)*** for their $1315.65\ldots$ and their $187.564\ldots$ correctly substituted in formula.

$\mathrm{HC} = 1330\ldots\ (\mathrm{m}) \quad (1328.95\ldots)$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from their answer to parts (a) and (f).

Examiners report

a. Most candidates were able to recognize and use the cosine rule correctly in part (a) and then to complete part (b) – though perhaps not giving the answer to the correct level of accuracy. It is expected that candidates can use “distance = speed x time” without the formula being given. The work involving sine rule was less successful, though correct responses were given by the great majority and the area of the course was again successfully completed by most candidates. A common error throughout these parts was the use of the total length of the course. A more fundamental error was the halving of the angle and/or the base in calculations – this error has been seen in a number of sessions and perhaps needs more emphasis.

b. Most candidates were able to recognize and use the cosine rule correctly in part (a) and then to complete part (b) – though perhaps not giving the answer to the correct level of accuracy. It is expected that candidates can use “distance = speed x time” without the formula being given. The work involving sine rule was less successful, though correct responses were given by the great majority and the area of the course was again successfully completed by most candidates. A common error throughout these parts was the use of the total length of the course. A more fundamental error was the halving of the angle and/or the base in calculations – this error has been seen in a number of sessions and perhaps needs more emphasis.

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d. Most candidates were able to recognize and use the cosine rule correctly in part (a) and then to complete part (b) – though perhaps not giving the answer to the correct level of accuracy. It is expected that candidates can use “distance = speed x time” without the formula being given. The work involving sine rule was less successful, though correct responses were given by the great majority and the area of the course was again successfully completed by most candidates. A common error throughout these parts was the use of the total length of the course. A more fundamental error was the halving of the angle and/or the base in calculations – this error has been seen in a number of sessions and perhaps needs more emphasis.

e. In part (e), unless evidence was presented, reasoning marks did not accrue; the interpretative nature of this part was a significant discriminator in determining the quality of a response.

f. There were many instances of parts (f) and (g) being left blank and angle of elevation is still not well understood. Again, the interpretative nature of part (g) – even when part (f) was correct – caused difficulties

g. There were many instances of parts (f) and (g) being left blank and angle of elevation is still not well understood. Again, the interpretative nature of part (g) – even when part (f) was correct – caused difficulties

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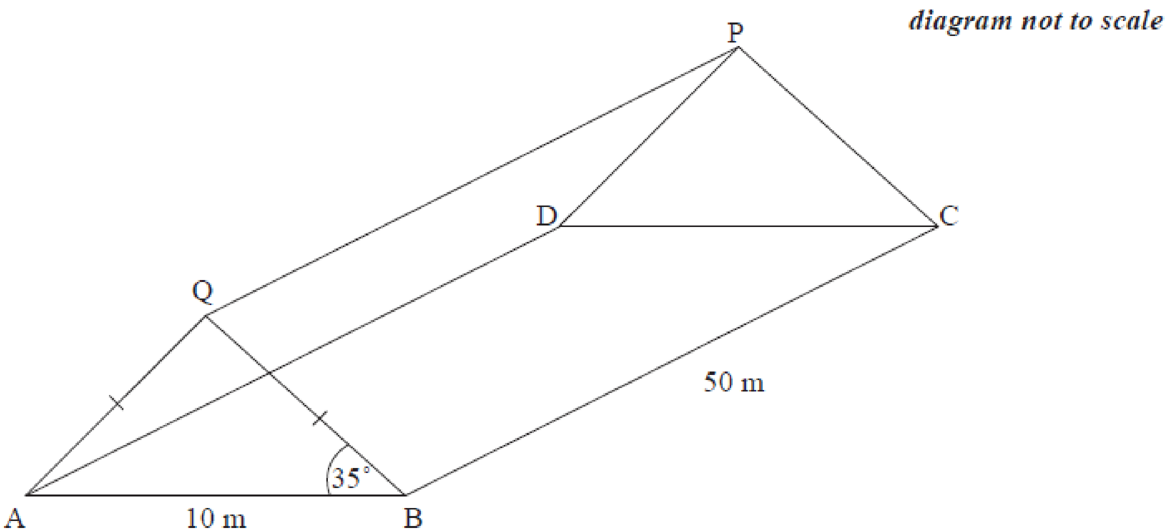
A greenhouse ABCDPQ is constructed on a rectangular concrete base ABCD and is made of glass. Its shape is a right prism, with cross section, ABQ, an isosceles triangle. The length of BC is 50 m, the length of AB is 10 m and the size of angle QBA is 35°.



a. Write down the size of angle AQB. [1]

b. Calculate the length of AQ. [3]

c. Calculate the length of AC. [2]

d. Show that the length of CQ is 50.37 m, correct to 4 significant figures. [2]

e. Find the size of the angle AQC. [3]

f. Calculate the total area of the glass needed to construct [5]

(i) the two rectangular faces of the greenhouse;

(ii) the two triangular faces of the greenhouse.

g. The cost of one square metre of glass used to construct the greenhouse is 4.80 USD. [3]

Calculate the cost of glass to make the greenhouse. Give your answer correct to the nearest 100 USD.

Markscheme

a. 110° ***(A1)***

b. $\frac{AQ}{\sin 35^\circ} = \frac{10}{\sin 110^\circ}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted sine rule formula, ***(A1)*** for their correct substitutions.

OR

$AQ = \frac{5}{\cos 35°}$ ***(A1)(M1)***

Note: Award ***(A1)*** for 5 seen, ***(M1)*** for correctly substituted trigonometric ratio.

$AQ = 6.10$ (6.10387...) ***(A1)*(ft)*(G2)***

Notes: Follow through from their answer to part (a).

c. $AC^2 = 10^2 + 50^2$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted Pythagoras formula.

$AC = 51.0(\sqrt{2600}, 50.9901...)$ ***(A1)(G2)***

d. $QC^2 = (6.10387...)^2 + (50)^2$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted Pythagoras formula.

$QC = 50.3711...$ ***(A1)***

$= 50.37$ ***(AG)***

Note: Both the unrounded and rounded answers must be seen to award ***(A1)***.

If 6.10 is used then 50.3707... is the unrounded answer.

For an incorrect follow through from part (b) award a maximum of ***(M1)(A0)*** – the given answer must be reached to award the final ***(A1)(AG)***.

e. $\cos AQC = \frac{(6.10387...)^2 + (50.3711...)^2 - (50.9901...)^2}{2(6.10387...)(50.3711...)}$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substituted cosine rule formula, ***(A1)*(ft)** for their correct substitutions.

= 92.4° $(92.3753...°)$ ***(A1)*(ft)*(G2)***

Notes: Follow through from their answers to parts (b), (c) and (d). Accept 92.2 if the 3 sf answers to parts (b), (c) and (d) are used.

Accept 92.5° $(92.4858...°)$ if the 3 sf answers to parts (b), (c) and 4 sf answers to part (d) used.

f. (i) $2(50 \times 6.10387...)$ ***(M1)***

Note: Award ***(M1)*** for their correctly substituted rectangular area formula, the area of one rectangle is not sufficient.

= 610 m^2 (610.387...) ***(A1)*(ft)*(G2)***

Notes: Follow through from their answer to part (b).

The answer is 610 m^2. The units are required.

(ii) Area of triangular face $= \frac{1}{2} \times 10 \times 6.10387... \times \sin 35°$ ***(M1)(A1)*(ft)**

OR

Area of triangular face $= \frac{1}{2} \times 6.10387... \times 6.10387... \times \sin 110^\circ$ ***(M1)(A1)*(ft)**

$= 17.5051...$

Note: Award ***(M1)*** for substituted triangle area formula, ***(A1)*(ft)** for correct substitutions.

OR

(Height of triangle) $= (6.10387...)^2 - 5^2$

$= 3.50103...$

Area of triangular face $= \frac{1}{2} \times 10 \times their\ height$

$= 17.5051...$

Note: Award ***(M1)*** for substituted triangle area formula, ***(A1)*(ft)** for correctly substituted area formula. If 6.1 is used, the height is 3.49428... and the area of both triangular faces 34.9 m^2

Area of both triangular faces = 35.0 m^2 (35.0103...) ***(A1)(ft)(G2)***

Notes: The answer is 35.0 m^2. The units are required. Do not penalize if already penalized in part (f)(i). Follow through from their part (b).

g. (610.387... + 35.0103...) × 4.80 ***(M1)***

= 3097.90... ***(A1)*(ft)**

Notes: Follow through from their answers to parts (f)(i) and (f)(ii).

Accept 3096 if the 3 sf answers to part (f) are used.

= 3100 ***(A1)*(ft)*(G2)***

Notes: Follow through from their unrounded answer, irrespective of whether it is correct. Award ***(M1)(A2)*** if working is shown and 3100 seen without the unrounded answer being given.

Examiners report

a. Most candidates used the appropriate area formula – however, some did not read the question with the attention it required and found the area of three rectangles – one of which being the stated "concrete base".

b. Most candidates used the appropriate area formula – however, some did not read the question with the attention it required and found the area of three rectangles – one of which being the stated "concrete base".

c. Most candidates were able to recognize sine rule, substitute correctly and reach the required result.

d. Most candidates were able to recognize sine rule, substitute correctly and reach the required result. The use of Pythagoras' theorem was also successful, the major source of error being the lack of unrounded and rounded answers in part (d).

Again, most candidates used the appropriate area formula – however, some did not read the question with the attention it required and found the area of three rectangles – one of which being the stated "concrete base".

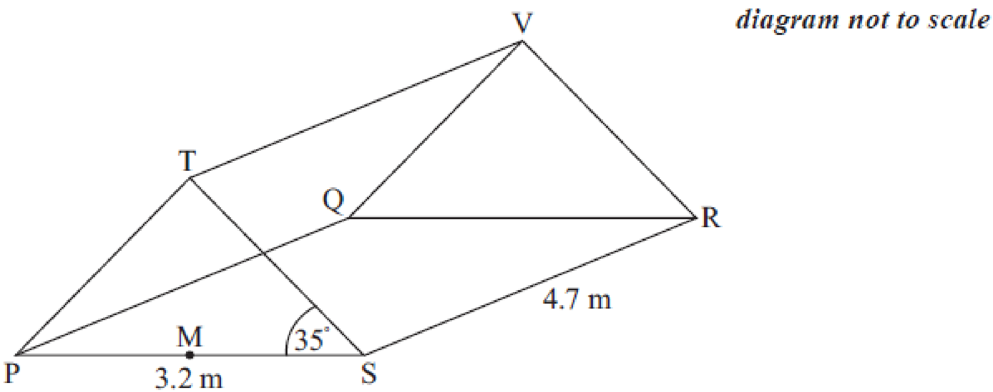
e. Most candidates were able to recognize sine rule, substitute correctly and reach the required result. Part (e) was less well answered, due in part to the triangle being in three dimensions. However, all three sides had either been asked for in previous parts or given and all that was required was a sketch of a triangle with the vertices labelled; such a diagram was never on any script and this technique should be encouraged.

Again, most candidates used the appropriate area formula – however, some did not read the question with the attention it required and found the area of three rectangles – one of which being the stated "concrete base".

f. Most candidates used the appropriate area formula – however, some did not read the question with the attention it required and found the area of three rectangles – one of which being the stated "concrete base".

g. Most candidates used the appropriate area formula – however, some did not read the question with the attention it required and found the area of three rectangles – one of which being the stated "concrete base".

A tent is in the shape of a triangular right prism as shown in the diagram below.



The tent has a rectangular base PQRS .

PTS and QVR are isosceles triangles such that PT = TS and QV = VR .

PS is 3.2 m , SR is 4.7 m and the angle TSP is 35 °.

a. Show that the length of side ST is 1.95 m, correct to 3 significant figures. [3]

b. Calculate the area of the triangle PTS. [3]

c. Write down the area of the rectangle STVR. [1]

d. Calculate the **total** surface area of the tent, including the base. [3]

e. Calculate the volume of the tent. [2]

f. A pole is placed from V to M, the midpoint of PS. [4]

Find in metres,

(i) the height of the tent, TM;

(ii) the length of the pole, VM.

g. Calculate the angle between VM and the base of the tent. [2]

Markscheme

a. $\mathrm{ST} = \frac{1.6}{\cos 35^\circ}$ ***(M1)(A1)***

Note: Award ***(M1)*** for correctly substituted trig equation, ***(A1)*** for 1.6 seen.

OR

$\frac{\mathrm{ST}}{\sin 35^\circ} = \frac{3.2}{\sin 110^\circ}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted sine rule equation, ***(A1)*** for correct substitutions.

ST = 1.95323... ***(A1)***

= 1.95 (m) ***(AG)***

Notes: Both unrounded and rounded answer must be seen for final ***(A1)*** to be awarded.

b. $\frac{1}{2} \times 3.2 \times 1.95323... \times \sin 35^\circ$ **or** $\frac{1}{2} \times 1.95323... \times 1.95323... \times \sin 110^\circ$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted area formula, ***(A1)*** for correct substitutions. Do not award follow through marks.

= 1.79 m^2 (1.79253...m^2) ***(A1)(G2)***

Notes: The answer is 1.79 m^2, **units are required**. Accept 1.78955... from using 1.95.

OR

$\frac{1}{2} \times 3.2 \times 1.12033...$ ***(A1) (M1)***

Note: Award ***(A1)*** for the correct value for TM (1.12033...) **OR** correct expression for TM (i.e. 1.6tan35°, $\sqrt{(1.95323...)^2 - 1.6^2}$), ***(M1)*** for correctly substituted formula for triangle area.

= 1.79 m^2 (1.79253...m^2) ***(A1)(G2)***

Notes: The answer is 1.79 m^2, **units are required**. Accept 1.78 m^2 from using 1.95.

c. 9.18 m^2 (9.18022 m^2) ***(A1)(G1)***

Notes: The answer is 9.18 m^2, **units are required**. Do not penalize if lack of units was already penalized in (b). Do not award follow through marks here. Accept 9.17 m^2 (9.165 m^2) from using 1.95.

d. $2 \times 1.79253... + 2 \times 9.18022... + 4.7 \times 3.2$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for addition of three products, ***(A1)*****(ft)** for three correct products.

= 37.0 m^2 (36.9855...m^2) ***(A1)*****(ft)*****(G2)***

Notes: The answer is 37.0 m^2, **units are required**. Accept 36.98 m^2 from using 3sf answers. Follow through from their answers to (b) and (c). Do not penalize if lack of units was penalized earlier in the question.

e. $1.79253\ldots \times 4.7$ ***(M1)***

Note: Award ***(M1)*** for their correctly substituted volume formula.

= 8.42 m^3 (8.42489...m^3) ***(A1)*(ft)*(G2)***

Notes: The answer is 8.42 m^3, **units are required**. Accept 8.41 m^3 from use of 1.79. An answer of 8.35, from use of TM = 1.11, will receive follow-through marks if working is shown. Follow through from their answer to part (b). Do not penalize if lack of units was penalized earlier in the question.

f. (i) $\mathrm{TM} = 1.6 \tan 35^\circ$ ***(M1)***

Notes: Award ***(M1)*** for their correct substitution in trig ratio.

OR

$\mathrm{TM} = \sqrt{(1.95323\ldots)^2 - 1.6^2}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in Pythagoras' theorem.

OR

$\frac{3.2 \times \mathrm{TM}}{2} = 1.79253\ldots$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution in area of triangle formula.

= 1.12 (m) (1.12033...) ***(A1)*(ft)*(G2)***

Notes: Follow through from their answer to (b) if area of triangle is used. Accept 1.11 (1.11467) from use of ST = 1.95.

(ii) $\mathrm{VM} = \sqrt{1.12033\ldots^2 + 4.7^2}$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution in Pythagoras' theorem.

= 4.83 (m) (4.83168) ***(A1)*(ft)*(G2)***

Notes: Follow through from (f)(i).

g. $\sin^{-1}\left(\frac{1.12033\ldots}{4.83168\ldots}\right)$ ***(M1)***

OR

$\cos^{-1}\left(\frac{4.7}{4.83168\ldots}\right)$ ***(M1)***

OR

$\tan^{-1}\left(\frac{1.12033\ldots}{4.7}\right)$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted trig equation.

OR

$\cos^{-1}\left(\frac{4.7^2+(4.83168\ldots)^2-(1.12033\ldots)^2}{2\times4.7\times4.83168\ldots}\right)$ ***(M1)***

Note: Award ***(M1)*** for correctly substituted cosine formula.

= 13.4° (13.4073...) ***(A1)*(ft)*(G2)***

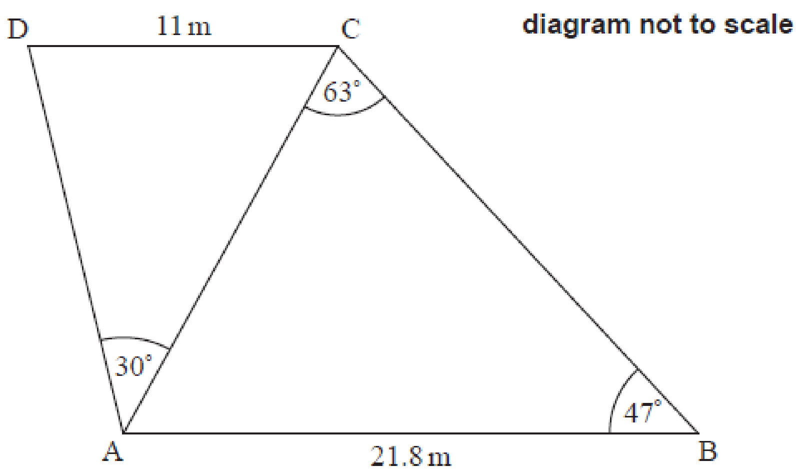
Notes: Accept 13.3°. Follow through from part (f).

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]

---

a. A playground, when viewed from above, is shaped like a quadrilateral, ABCD, where $\mathrm{AB} = 21.8\,\mathrm{m}$ and $\mathrm{CD} = 11\,\mathrm{m}$. Three of the internal angles have been measured and angle $\mathrm{ABC} = 47^\circ$, angle $\mathrm{ACB} = 63^\circ$ and angle $\mathrm{CAD} = 30^\circ$. This information is represented in the following diagram. [3]



Calculate the distance AC.

b. Calculate angle ADC. [3]

c. There is a tree at C, perpendicular to the ground. The angle of elevation to the top of the tree from D is 35°. [2]

Calculate the height of the tree.

d. Chavi estimates that the height of the tree is $6\,\mathrm{m}$. [2]

Calculate the percentage error in Chavi’s estimate.

e. Chavi is celebrating her birthday with her friends on the playground. Her mother brings a $2\ \mathrm{litre}$ bottle of orange juice to share among them. [3]

She also brings **cone-shaped** paper cups.

Each cup has a vertical height of $10\,\mathrm{cm}$ and the top of the cup has a diameter of $6\,\mathrm{cm}$.

Calculate the volume of one paper cup.

f. Calculate the maximum number of cups that can be completely filled with the 2 litre bottle of orange juice. [3]

Markscheme

a. $\frac{21.8}{\sin 63^\circ} = \frac{\text{AC}}{\sin 47^\circ}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into the sine rule formula, ***(A1)*** for correct substitution.

$(\text{AC} =)\ 17.9\,(\text{m})\ \ (17.8938...\,(\text{m}))$ ***(A1)(G2)***

b. $\frac{11}{\sin 30} = \frac{17.8938...}{\sin \text{ADC}}$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substitution into the sine rule formula, ***(A1)*** for correct substitution.

$(\text{Angle ADC} =)\ \ 54.4^\circ\ \ (54.4250...^\circ)$ ***(A1)*****(ft)*****(G2)***

Note: Accept 54.5 $(54.4527...)$ or 126 $(125.547...)$ from using their 3 sf answer.
Follow through from part (a). Accept $125.575...$

c. $11 \times \tan 35^\circ$ (or equivalent) ***(M1)***

Note: Award ***(M1)*** for correct substitution into trigonometric ratio.

$7.70\,(\text{m})\ \ (7.70228...\,(\text{m}))$ ***(A1)(G2)***

d. $\left|\frac{6-7.70228...}{7.70228...}\right| \times 100\,\%$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the percentage error formula.

OR

$100 - \left|\frac{6\times 100}{7.70228...}\right|$ ***(M1)***

Note: Award ***(M1)*** for the alternative method.

$22.1\,(\%)\ \ (22.1009...\,(\%))$ ***(A1)*****(ft)*****(G2)***

Note: Award at most ***(M1)(A0)*** for a final answer that is negative. Follow through from part (c).

e. $\frac{1}{3}\pi \times 3^2 \times 10$ ***(A1)(M1)***

Note: Award ***(A1)*** for 3 seen, ***(M1)*** for their correct substitution into volume of a cone formula.

$94.2\,\text{cm}^3\ \ (30\pi\,\text{cm}^3,\ 94.2477...\,\text{cm}^3)$ ***(A1)(G3)***

Note: The answer is $94.2\,\text{cm}^3$, units are required. Award at most ***(A0)(M1)(A0)*** if an incorrect value for r is used.

f. $\frac{2000}{94.2477...}$ **OR** $\frac{2}{0.0942477...}$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct conversion (litres to cm^3 or cm^3 to litres), ***(M1)*** for dividing by their part (e) (or their converted part (e)).

21 ***(A1)*****(ft)*****(G2)***

Note: The final ***(A1)*** is not awarded if the final answer is not an integer. Follow through from part (e), but only if the answer is rounded **down**.

Examiners report

a. Question 4: Trigonometry and volumes of 3D solids

This question was done well by most candidates. Trigonometry was a real strength with competent use of the sine rule. A small minority treated CB as parallel to AB and hence used alternate angles. The lack of a diagram in part (c) held some candidates back as they struggled to form the correct trigonometric ratio. Percentage error in part (d) was generally good. Most candidates scored the two marks as their answer to part (c) was followed through in part (d). Some candidates are still giving negative answers to percentage error problems. The common mistake in this part was the use of the new value in the denominator rather than the original value. Part (f) was less successful, in general, with a number of candidates not able to do the conversion.

b. Question 4: Trigonometry and volumes of 3D solids

   This question was done well by most candidates. Trigonometry was a real strength with competent use of the sine rule. A small minority treated CB as parallel to AB and hence used alternate angles. The lack of a diagram in part (c) held some candidates back as they struggled to form the correct trigonometric ratio. Percentage error in part (d) was generally good. Most candidates scored the two marks as their answer to part (c) was followed through in part (d). Some candidates are still giving negative answers to percentage error problems. The common mistake in this part was the use of the new value in the denominator rather than the original value. Part (f) was less successful, in general, with a number of candidates not able to do the conversion.

c. Question 4: Trigonometry and volumes of 3D solids

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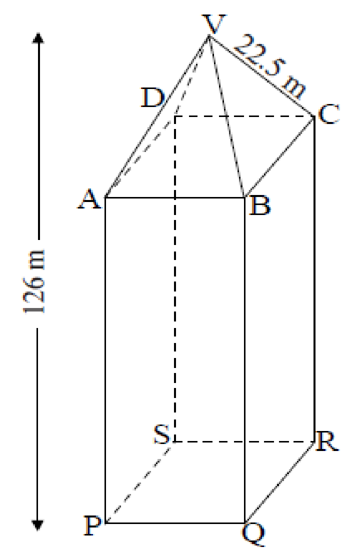
The diagram shows an office tower of total height 126 metres. It consists of a square based pyramid VABCD on top of a cuboid ABCDPQRS.

V is directly above the centre of the base of the office tower.

The length of the sloping edge VC is 22.5 metres and the angle that VC makes with the base ABCD (angle VCA) is 53.1°.

diagram not to scale



a.i. Write down the length of VA in metres. [1]

a.ii. Sketch the triangle VCA showing clearly the length of VC and the size of angle VCA. [1]

b. Show that the height of the pyramid is 18.0 metres correct to 3 significant figures. [2]

c. Calculate the length of AC in metres. [3]

d. Show that the length of BC is 19.1 metres correct to 3 significant figures. [2]

e. Calculate the volume of the tower. [4]

f. To calculate the cost of air conditioning, engineers must estimate the weight of air in the tower. They estimate that 90 % of the volume of the tower is occupied by air and they know that 1 m^3 of air weighs 1.2 kg. [3]

Calculate the weight of air in the tower.

Markscheme

a.i. 22.5 (m) ***(A1)***

[1 mark]

a.ii. onbekend.png ***(A1)***

[1 mark]

b. h = 22.5 sin 53.1° ***(M1)***

= 17.99 ***(A1)***

= 18.0 ***(AG)***

Note: Unrounded answer must be seen for ***(A1)*** to be awarded.

Accept 18 as ***(AG)***.

[2 marks]

c. $\mathrm{AC} = 2\sqrt{22.5^2 - 17.99...^2}$ ***(M1)(M1)***

Note: Award ***(M1)*** for multiplying by 2, ***(M1)*** for correct substitution into formula.

OR

AC = 2(22.5)cos53.1° ***(M1)(M1)***

Notes: Award ***(M1)*** for correct use of cosine trig ratio, ***(M1)*** for multiplying by 2.

OR

$AC^2 = 22.5^2 + 22.5^2 - 2(22.5)(22.5)\cos 73.8°$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted cosine formula, ***(A1)*** for correct substitutions.

OR

$\frac{\mathrm{AC}}{\sin(73.8^\circ)} = \frac{22.5}{\sin(53.1^\circ)}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted sine formula, ***(A1)*** for correct substitutions.

AC = 27.0 ***(A1)(G2)***

[3 marks]

d. $\mathrm{BC} = \sqrt{13.5^2 + 13.5^2}$ ***(M1)***

= 19.09 ***(A1)***

= 19.1 ***(AG)***

OR

$x^2 + x^2 = 27^2$ ***(M1)***

$2x^2 = 27^2$ ***(A1)***

BC = 19.09… ***(A1)***

= 19.1 ***(AG)***

Notes: Unrounded answer must be seen for ***(A1)*** to be awarded.

[2 marks]

e. Volume = Pyramid + Cuboid

$= \frac{1}{3}(18)(19.1^2) + (108)(19.1^2)$ ***(A1)(M1)(M1)***

Note: Award ***(A1)*** for 108, the height of the cuboid seen. Award ***(M1)*** for correctly substituted volume of cuboid and ***(M1)*** for correctly substituted volume of pyramid.

= $41\,588$ *(41 553 if* $2(13.5^2)$ *is used)*

= $41\,600$ m^3 ***(A1)*(ft)*(G3)***

[4 marks]

f. Weight of air = $41\,600 \times 1.2 \times 0.9$ ***(M1)(M1)***

= $44\,900$ kg ***(A1)*(ft)*(G2)***

Note: Award ***(M1)*** for their part (e) × 1.2, ***(M1)*** for × 0.9.

Award at most ***(M1)(M1)(A0)*** if the volume of the cuboid is used.

[3 marks]

Examiners report

a.i. This question also caused many problems for the candidature. There seems to be a lack of ability in visualising a problem in three dimensions – clearly, further exposure to such problems is needed by the students. Further, as in question 2, the final two parts of the question were independent of those preceding them; many candidates did not reach these parts, though for some, these were the only parts of the question attempted. There is also a lack of awareness of the appropriate volume formula on the formula sheet to use.

a.ii. This question also caused many problems for the candidature. There seems to be a lack of ability in visualising a problem in three dimensions – clearly, further exposure to such problems is needed by the students. Further, as in question 2, the final two parts of the question were independent of those preceding them; many candidates did not reach these parts, though for some, these were the only parts of the question attempted. There is also a lack of awareness of the appropriate volume formula on the formula sheet to use.

b. This question also caused many problems for the candidature. There seems to be a lack of ability in visualising a problem in three dimensions – clearly, further exposure to such problems is needed by the students. Further, as in question 2, the final two parts of the question were independent of those preceding them; many candidates did not reach these parts, though for some, these were the only parts of the question attempted. There is also a lack of awareness of the appropriate volume formula on the formula sheet to use.

c. This question also caused many problems for the candidature. There seems to be a lack of ability in visualising a problem in three dimensions – clearly, further exposure to such problems is needed by the students. Further, as in question 2, the final two parts of the question were independent of those preceding them; many candidates did not reach these parts, though for some, these were the only parts of the question attempted. There is also a lack of awareness of the appropriate volume formula on the formula sheet to use.

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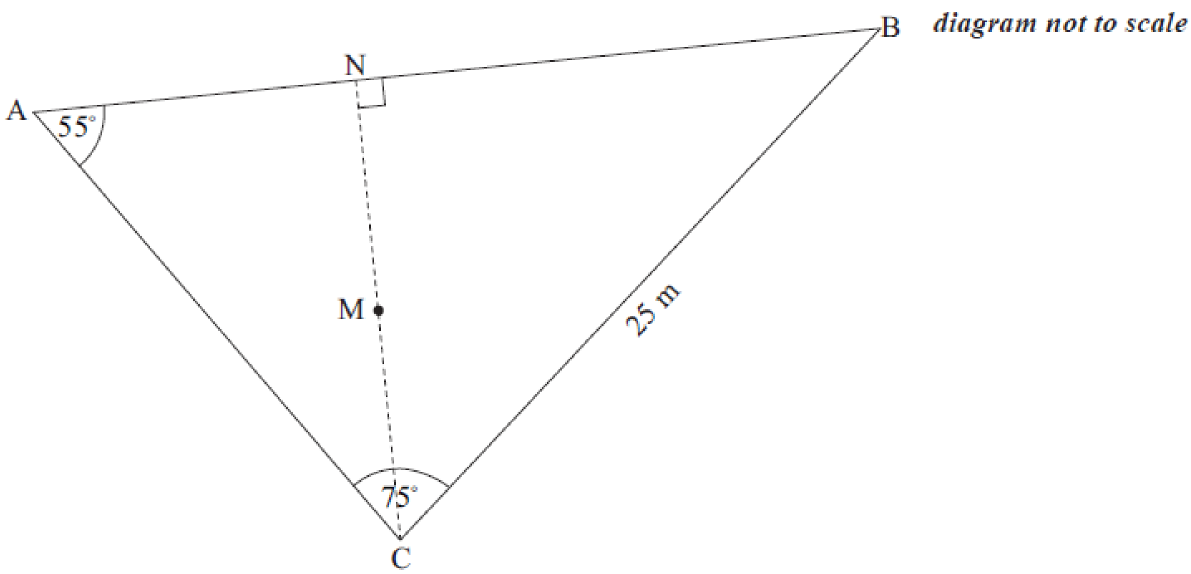
The diagram represents a small, triangular field, ABC , with $\mathrm{BC} = 25\ \mathrm{m}$, angle $\mathrm{BAC} = 55^\circ$ and angle $\mathrm{ACB} = 75^\circ$.



a. Write down the size of angle ABC. [1]

b. Calculate the length of AC. [3]

c. Calculate the area of the field ABC. [3]

d. N is the point on AB such that CN is perpendicular to AB. M is the midpoint of CN. [3]

Calculate the length of NM.

e. A goat is attached to one end of a rope of length 7 m. The other end of the rope is attached to the point M. [5]

Decide whether the goat can reach point P, the midpoint of CB. Justify your answer.

Markscheme

a. $\text{Angle ABC} = 50^\circ$ ***(A1)***

[1 mark]

b. $\frac{\text{AC}}{\sin 50^\circ} = \frac{25}{\sin 55^\circ}$ ***(M1)(A1)*****(ft)**

Notes: Award ***(M1)*** for substitution into the correct formula, ***(A1)*****(ft)** for correct substitution. Follow through from their angle ABC.

$\text{AC} = 23.4 \text{ m}$ ***(A1)*****(ft)*****(G2)***

[3 marks]

c. $\text{Area of } \Delta \text{ ABC} = \frac{1}{2} \times 23.379\ldots \times 25 \times \sin 75^\circ$ ***(M1)(A1)*****(ft)**

Notes: Award ***(M1)*** for substitution into the correct formula, ***(A1)*****(ft)** for correct substitution. Follow through from their AC.

OR

$\text{Area of triangle ABC} = \frac{29.479\ldots \times 19.151\ldots}{2}$ ***(A1)*****(ft)*****(M1)***

Note: ***(A1)*****(ft)** for correct values of AB (29.479…) and CN (19.151…). Follow through from their (a) and /or (b). Award ***(M1)*** for substitution of their values of AB and CN into the correct formula.

$\text{Area of } \Delta \text{ ABC} = 282 \text{ m}^2$ ***(A1)*****(ft)*****(G2)***

Note: Accept 283 m^2 if 23.4 is used.

[3 marks]

d. $\text{NM} = \frac{25 \times \sin 50^\circ}{2}$ ***(M1)(M1)***

Note: Award ***(M1)*** for $25 \times \sin 50^\circ$ or equivalent for the length of CN. ***(M1)*** for dividing their CN by 2.

$\text{NM} = 9.58 \text{ m}$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from their angle ABC.

Notes: Premature rounding of CN leads to the answers 9.60 or 9.6. Award at most ***(M1)(M1)(A0)*** if working seen. Do not penalize with ***(AP)***. CN may be found in (c).

Note: The working for this part of the question may be in part (b).

[3 marks]

e. $\text{Angle NCB} = 40^\circ$ seen ***(A1)*****(ft)**

Note: Follow through from their (a).

From triangle MCP:

$\mathrm{MP}^2 = (9.5756\ldots)^2 + 12.5^2 - 2 \times 9.5756\ldots \times 12.5 \times \cos(40^\circ)$ ***(M1)(A1)*(ft)**

$\mathrm{MP} = 8.034\ldots\ \mathrm{m}$ ***(A1)*(ft)*(G3)***

Notes: Award ***(M1)*** for substitution into the correct formula, ***(A1)*(ft)** for their correct substitution. Follow through from their d). Award ***(G3)*** for correct value of MP seen without working.

OR

From right triangle MCP

$\mathrm{CP} = 12.5\ \mathrm{m}$ seen ***(A1)***

$\mathrm{MP}^2 = (12.5)^2 - (9.575\ldots)^2$ ***(M1)(A1)*(ft)**

$\mathrm{MP} = 8.034\ldots\ \mathrm{m}$ ***(A1)(G3)*(ft)**

Notes: Award ***(M1)*** for substitution into the correct formula, ***(A1)*(ft)** for their correct substitution. Follow through from their (d). Award ***(G3)*** for correct value of MP seen without working.

OR

From right triangle MCP

$\text{Angle MCP} = 40^\circ$ seen ***(A1)*(ft)**

$\frac{\mathrm{MP}}{12.5} = \sin(40^\circ)$ or equivalent ***(M1)(A1)*(ft)**

$\mathrm{MP} = 8.034\ldots\ \mathrm{m}$ ***(A1)(G3)*(ft)**

Notes: Award ***(M1)*** for substitution into the correct formula, ***(A1)*(ft)** for their correct substitution. Follow through from their (a). Award ***(G3)*** for correct value of MP seen without working.

The goat cannot reach point P as $\mathrm{MP} > 7\ \mathrm{m}$. ***(A1)*(ft)**

Note: Award ***(A1)*(ft)** only if their value of MP is compared to $7\ \mathrm{m}$, and conclusion is stated.

[5 marks]

Examiners report

a. Many candidates assumed incorrectly that the triangle ABC is isosceles or/and that CN is an angle bisector, and those assumptions led them to use incorrect methods. Wherever those assumptions were made first, all or most of the marks were lost in that specific part of Question 4. There were provisions in the mark scheme to follow through in subsequent parts. Most candidates at least attempted parts a), b) and c). Some candidates incorrectly used an area formula for a right triangle in c) and lost all marks. Many candidates lost a mark for premature rounding in d). Part e) proved to be especially difficult for the candidates. Here many candidates offered guesses instead of sound mathematical reasoning.

b. Many candidates assumed incorrectly that the triangle ABC is isosceles or/and that CN is an angle bisector, and those assumptions led them to use incorrect methods. Wherever those assumptions were made first, all or most of the marks were lost in that specific part of Question 4. There were provisions in the mark scheme to follow through in subsequent parts. Most candidates at least attempted parts a), b) and c). Some candidates incorrectly used an area formula for a right triangle in c) and lost all marks. Many candidates lost a mark for premature rounding in d). Part e) proved to be especially difficult for the candidates. Here many candidates offered guesses instead of sound mathematical reasoning.

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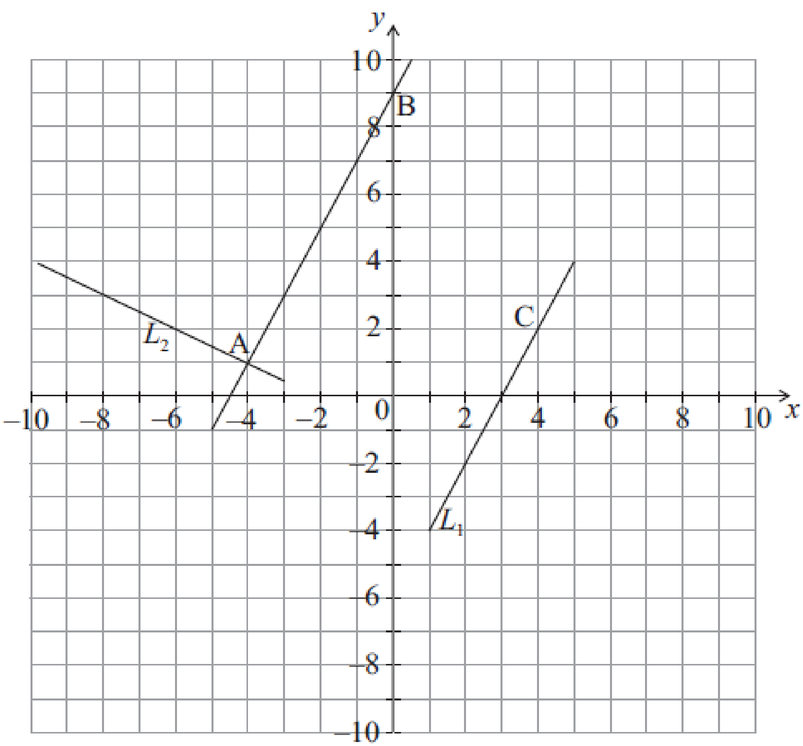
The points A (−4, 1), B (0, 9) and C (4, 2) are plotted on the diagram below. The diagram also shows the lines AB, L_1 and L_2.



a. Find the gradient of AB. [2]

b. L_1 passes through C and is parallel to AB. [1]

Write down the y-intercept of L_1.

c. L_2 passes through A and is perpendicular to AB. [3]

Write down the equation of L_2. Give your answer in the form ***ax + by + d* = 0** where a, b and $d \in \mathbb{Z}$.

d. Write down the coordinates of the point D, the intersection of L_1 and L_2. [1]

e. There is a point R on L_1 such that ABRD is a rectangle. [2]

Write down the coordinates of R.

f. The distance between A and D is $\sqrt{45}$. [4]

(i) Find the distance between D and R .

(ii) Find the area of the triangle BDR .

Markscheme

a. $\frac{9-1}{0-(-4)}$ ***(M1)***

= 2 ***(A1)(G2)***

Notes: Award ***(M1)*** for correct substitution into the gradient formula.

[2 marks]

b. –6 ***(A1)***

Note: Accept (0, –6) .

[1 mark]

c. $y = -\frac{1}{2}x - 1$ (or equivalent) ***(A1)*(ft)*(A1)***

Notes: Award ***(A1)*(ft)** for gradient, ***(A1)*** for correct y-intercept. Follow through from their gradient in (a).

$x + 2y + 2 = 0$ ***(A1)*(ft)**

Notes: Award ***(A1)*(ft)** from their gradient and their y-intercept. Accept any multiple of this equation with integer coefficients.

OR

$y - 1 = -\frac{1}{2}(x + 4)$ (or equivalent) ***(A1)*(ft)*(A1)***

Note: Award ***(A1)*(ft)** for gradient, ***(A1)*** for any point on the line correctly substituted in equation.

$x + 2y + 2 = 0$ ***(A1)*(ft)**

Notes: Award ***(A1)*(ft)** from their equation. Accept any multiple of this equation with integer coefficients.

[3 marks]

d. D(2, –2) or $x = 2$, $y = –2$ ***(A1)***

Note: Award ***(A0)*** if brackets not present.

[1 mark]

e. R(6, 6) **or** $x = 6$, $y = 6$ ***(A1)(A1)***

Note: Award at most ***(A0)(A1)*****(ft)** if brackets not present and absence of brackets has not already been penalised in part (d).

[2 marks]

f. (i) $\mathrm{DR} = \sqrt{8^2 + 4^2}$ ***(M1)***

$\mathrm{DR} = \sqrt{80}$ (8.94) ***(A1)*****(ft)*****(G2)***

Note: Award ***(M1)*** for correct substitution into the distance formula. Follow through from their D and R.

(ii) $\text{Area} = \frac{\sqrt{80}\times\sqrt{45}}{2}$ ***(M1)***

= 30 (30.0) ***(A1)*****(ft)*****(G2)***

Note: Award ***(M1)*** for correct substitution in the area of triangle formula. Follow through from their answer to part (f) (i).

[4 marks]

Examiners report

a. This question was in general well answered. In part (a) the gradient of the line AB was correctly found although some candidates did not substitute well in the gradient formula and found answers as $\frac{1}{2}$ or –2. Also some students read B as (0, 8) instead of (0, 9). In part (b) many students again did not make good use of time as they found the equation of the line instead of just extending it to find the y - intercept. The equation of L_2 in (c) was correctly found in the form $y = mx + c$ but very few students were able to rearrange the equation in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. In (d) many candidates found the coordinates of point D by solving simultaneous equations which led again to a waste of time. The last two parts of this question were well done by those students that attempted them.

b. This question was in general well answered. In part (a) the gradient of the line AB was correctly found although some candidates did not substitute well in the gradient formula and found answers as $\frac{1}{2}$ or –2. Also some students read B as (0, 8) instead of (0, 9). In part (b) many students again did not make good use of time as they found the equation of the line instead of just extending it to find the y - intercept. The equation of L_2 in (c) was correctly found in the form $y = mx + c$ but very few students were able to rearrange the equation in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. In (d) many candidates found the coordinates of point D by solving simultaneous equations which led again to a waste of time. The last two parts of this question were well done by those students that attempted them.

c. This question was in general well answered. In part (a) the gradient of the line AB was correctly found although some candidates did not substitute well in the gradient formula and found answers as $\frac{1}{2}$ or –2. Also some students read B as (0, 8) instead of (0, 9). In part (b) many students again did not make good use of time as they found the equation of the line instead of just extending it to find the y - intercept. The

equation of L_2 in (c) was correctly found in the form $y = mx + c$ but very few students were able to rearrange the equation in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. In (d) many candidates found the coordinates of point D by solving simultaneous equations which led again to a waste of time. The last two parts of this question were well done by those students that attempted them.

d. This question was in general well answered. In part (a) the gradient of the line AB was correctly found although some candidates did not substitute well in the gradient formula and found answers as $\frac{1}{2}$ or –2. Also some students read B as (0, 8) instead of (0, 9). In part (b) many students again did not make good use of time as they found the equation of the line instead of just extending it to find the y - intercept. The equation of L_2 in (c) was correctly found in the form $y = mx + c$ but very few students were able to rearrange the equation in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. In (d) many candidates found the coordinates of point D by solving simultaneous equations which led again to a waste of time. The last two parts of this question were well done by those students that attempted them.

e. This question was in general well answered. In part (a) the gradient of the line AB was correctly found although some candidates did not substitute well in the gradient formula and found answers as $\frac{1}{2}$ or –2. Also some students read B as (0, 8) instead of (0, 9). In part (b) many students again did not make good use of time as they found the equation of the line instead of just extending it to find the y - intercept. The equation of L_2 in (c) was correctly found in the form $y = mx + c$ but very few students were able to rearrange the equation in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. In (d) many candidates found the coordinates of point D by solving simultaneous equations which led again to a waste of time. The last two parts of this question were well done by those students that attempted them.

f. This question was in general well answered. In part (a) the gradient of the line AB was correctly found although some candidates did not substitute well in the gradient formula and found answers as $\frac{1}{2}$ or –2. Also some students read B as (0, 8) instead of (0, 9). In part (b) many students again did not make good use of time as they found the equation of the line instead of just extending it to find the y - intercept. The equation of L_2 in (c) was correctly found in the form $y = mx + c$ but very few students were able to rearrange the equation in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. In (d) many candidates found the coordinates of point D by solving simultaneous equations which led again to a waste of time. The last two parts of this question were well done by those students that attempted them.

Consider the functions $f(x) = \frac{2x+3}{x+4}$ and $g(x) = x + 0.5$.

a. Sketch the graph of the function $f(x)$, for $-10 \leqslant x \leqslant 10$. Indicating clearly the axis intercepts and any asymptotes. [6]

b. Write down the equation of the vertical asymptote. [2]

c. On the same diagram as part (a) sketch the graph of $g(x) = x + 0.5$. [2]

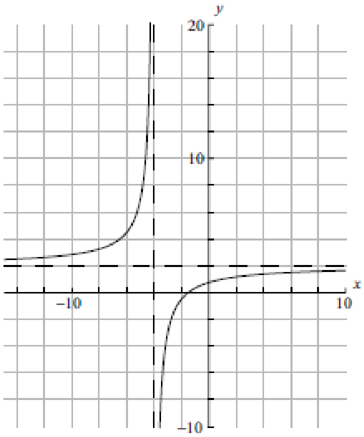
d. Using your graphical display calculator write down the coordinates of **one** of the points of intersection on the graphs of f and g, **giving your answer correct to five decimal places**. [3]

e. Write down the gradient of the line $g(x) = x + 0.5$. [1]

f. The line L passes through the point with coordinates $(-2, -3)$ and is perpendicular to the line $g(x)$. Find the equation of L. [3]

Markscheme

a.



(A6)

Notes: ***(A1)*** for labels and some idea of scale.
(A1) for x-intercept seen, ***(A1)*** for y-intercept seen in roughly the correct places (coordinates not required).
(A1) for vertical asymptote seen, ***(A1)*** for horizontal asymptote seen in roughly the correct places (equations of the lines not required).
(A1) for correct general shape.

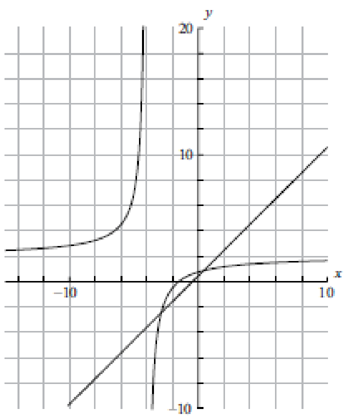
[6 marks]

b. $x = -4$ ***(A1)(A1)*****(ft)**

Note: ***(A1)*** for $x =$, ***(A1)*****(ft)** for -4.

[2 marks]

c.



(A1)(A1)

Note: ***(A1)*** for correct axis intercepts, ***(A1)*** for straight line

[2 marks]

d. $(-2.85078, -2.35078)$ OR $(0.35078, 0.85078)$ ***(G1)(G1)(A1)*****(ft)**

Notes: ***(A1)*** for x-coordinate, ***(A1)*** for y-coordinate, ***(A1)*****(ft)** for correct accuracy. Brackets required. If brackets not used award ***(G1)(G0)(A1)*****(ft)**.
Accept $x = -2.85078$, $y = -2.35078$ or $x = 0.35078$, $y = 0.85078$.

[3 marks]

e. $\text{gradient} = 1$ ***(A1)***

[1 mark]

f. $\text{gradient of perpendicular} = -1$ ***(A1)*****(ft)**

(can be implied in the next step)

$y = mx + c$

$-3 = -1 \times -2 + c$ ***(M1)***

$c = -5$

$y = -x - 5$ ***(A1)*(ft)*(G2)***

OR

$y + 3 = -(x + 2)$ ***(M1)(A1)*(ft)*(G2)***

Note: Award ***(G2)*** for correct answer with no working at all but ***(A1)(G1)*** if the gradient is mentioned as -1 then correct answer with no further working.

[3 marks]

Examiners report

a. This was not very well done. The graph was often correct but was so small that it was difficult to check if axes intercepts were correct or not. Often the vertical asymptote looked as if it were joined to the rest of the graph. Very few of the candidates put a scale and/or labels on their axes.

b. Reasonably well done. Some put $y = -4$ while others omitted the minus sign.

c. Fairly well done – but once again too small to check the axes intercepts properly. Also, many candidates did not appear to have a ruler to draw the straight line.

d. Well done.

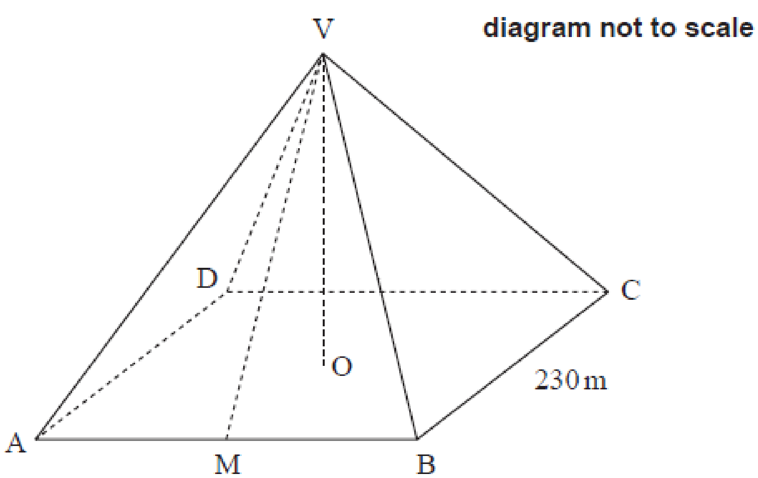
e. Most could find the gradient of the line.

f. Many forgot to find the gradient of the perpendicular line. Others had problems with the equation of a line in general.

---

a. The Great Pyramid of Giza in Egypt is a right pyramid with a square base. The pyramid is made of solid stone. The sides of the base are 230 m long. The diagram below represents this pyramid, labelled VABCD. [3]

V is the vertex of the pyramid. O is the centre of the base, ABCD . M is the midpoint of AB. Angle $\text{ABV} = 58.3°$.



Show that the length of VM is 186 metres, correct to three significant figures.

b. Calculate the height of the pyramid, VO . [2]

c. Find the volume of the pyramid. [2]

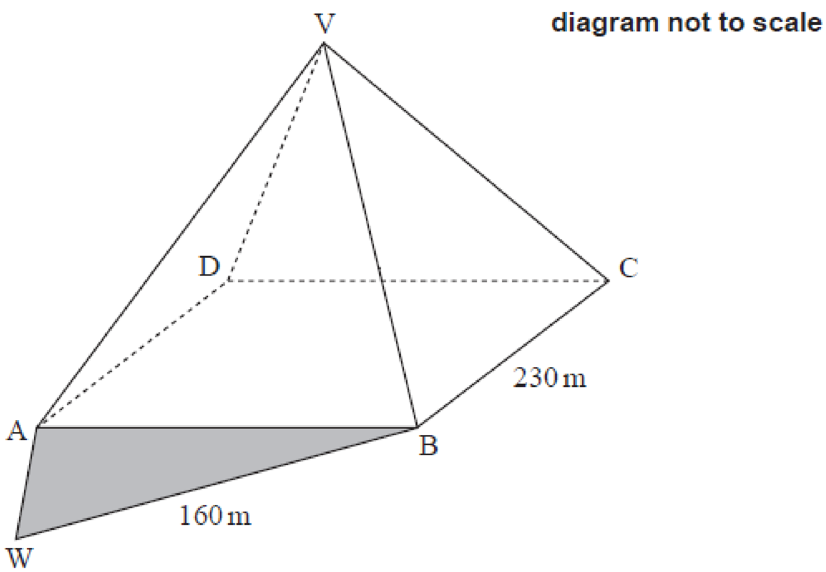
d. Write down your answer to part (c) in the form $a \times 10^k$ where $1 \leqslant a < 10$ and $k \in \mathbb{Z}$. [2]

e. Ahmad is a tour guide at the Great Pyramid of Giza. He claims that the amount of stone used to build the pyramid could build a wall 5 metres high and 1 metre wide stretching from Paris to Amsterdam, which are $430\,\mathrm{km}$ apart. [4]

Determine whether Ahmad's claim is correct. Give a reason.

f. Ahmad and his friends like to sit in the pyramid's shadow, ABW, to cool down. [6]

At mid-afternoon, $\mathrm{BW} = 160\,\mathrm{m}$ and angle $\mathrm{ABW} = 15^\circ$.



i) Calculate the length of AW at mid-afternoon.

ii) Calculate the area of the shadow, ABW, at mid-afternoon.

Markscheme

a. $\tan(58.3) = \frac{\mathrm{VM}}{115}$ **OR** $115 \times \tan(58.3^\circ)$ ***(A1)(M1)***

Note: Award ***(A1)*** for 115 $\left(ie\ \frac{230}{2}\right)$ seen, ***(M1)*** for correct substitution into trig formula.

$(\mathrm{VM} =)\ 186.200\,(\mathrm{m})$ ***(A1)***

$(\mathrm{VM} =)\ 186\,(\mathrm{m})$ ***(AG)***

Note: Both the rounded and unrounded answer must be seen for the final ***(A1)*** to be awarded.

b. $\mathrm{VO}^2 + 115^2 = 186^2$ **OR** $\sqrt{186^2 - 115^2}$ **(M1)**

Note: Award ***(M1)*** for correct substitution into Pythagoras formula. Accept alternative methods.

$(\mathrm{VO} =)\ 146\,(\mathrm{m})\ (146.188...)$ ***(A1)(G2)***

Note: Use of full calculator display for VM gives $146.443...\,(\mathrm{m})$.

c. **Units are required in part (c)**

$\frac{1}{3}(230^2 \times 146.188...)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in volume formula. Follow through from part (b).

$= 2\,580\,000\,\mathrm{m}^3\ \ (2\,577\,785...\,\mathrm{m}^3)$ ***(A1)*(ft)*(G2)***

Note: The answer is $2\,580\,000\,\mathrm{m}^3$, the units are required. Use of $\mathrm{OV} = 146.442$ gives $2582271...\,\mathrm{m}^3$

Use of $\mathrm{OV} = 146$ gives $2574466...\,\mathrm{m}^3$.

d. $2.58 \times 10^6\,(\mathrm{m}^3)$ ***(A1)*(ft)*(A1)*(ft)**

Note: Award ***(A1)*(ft)** for 2.58 and ***(A1)*(ft)** for $\times 10^6$.

Award ***(A0)(A0)*** for answers of the type: $2.58 \times 10^5\,(\mathrm{m}^3)$.

Follow through from part (c).

e. the volume of a wall would be $430\,000 \times 5 \times 1$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into volume formula.

$2150000\,(\mathrm{m}^3)$ ***(A1)(G2)***

which is less than the volume of the pyramid ***(R1)(ft)***

Ahmad is correct. ***(A1)(ft)***

OR

the length of the wall would be $\frac{\text{their part (c)}}{5\times1\times1000}$ ***(M1)***

Note: Award ***(M1)*** for dividing their part (c) by 5000.

$516\,(\mathrm{km})$ ***(A1)*(ft)*(G2)***

which is more than the distance from Paris to Amsterdam ***(R1)(ft)***

Ahmad is correct. ***(A1)(ft)***

Note: Do not award final ***(A1)*** without an explicit comparison. Follow through from part (c) or part (d). Award ***(R1)*** for reasoning that is consistent with their working in part (e); comparing two volumes, or comparing two lengths.

f. **Units are required in part (f)(ii).**

i) $\mathrm{AW}^2 = 160^2 + 230^2 - 2 \times 160 \times 230 \times \cos(15°)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into cosine rule formula, ***(A1)*** for correct substitution.

$\mathrm{AW} = 86.1\,(\mathrm{m})\ \ (86.0689...)$ ***(A1)(G2)***

Note: Award ***(M0)(A0)(A0)*** if BAW or AWB is considered to be a right angled triangle.

ii) $\text{area} = \frac{1}{2} \times 230 \times 160 \times \sin(15°)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into area formula, ***(A1)*** for correct substitutions.

$= 4760\,\mathrm{m}^2\ \ (4762.27...\,\mathrm{m}^2)$ ***(A1)(G2)***

Note: The answer is $4760\,\mathrm{m}^2$, the units are required.

Examiners report

a. Question 4: Trigonometry, volume and area.

   Many were able to write a correct trig ratio for part (a). The most common error was not to write the unrounded or the rounded answer. Some incorrectly used the given value of 186 in their proof. Part (b) was mostly answered correctly, with only a few candidates using Pythagoras' Theorem incorrectly. Most candidates used the correct formula to calculate the volume of the pyramid, but some did not find the correct area for the base of the pyramid. Some lost a mark for missing or for incorrect units. Even with an incorrect answer for part (c), candidates did very well on part (d). In part (e) some excellent justifications were given. However, many struggled to convert kilometres to metres, others were confused and compared surface area instead of volume. Some thought the volumes needed to be the same. For part (f) candidates often assumed a right angle at BAW or BWA. When they used the sine and cosine rule, this was mostly done correctly.

b. Question 4: Trigonometry, volume and area.

   Many were able to write a correct trig ratio for part (a). The most common error was not to write the unrounded or the rounded answer. Some incorrectly used the given value of 186 in their proof. Part (b) was mostly answered correctly, with only a few candidates using Pythagoras' Theorem incorrectly. Most candidates used the correct formula to calculate the volume of the pyramid, but some did not find the correct area for the base of the pyramid. Some lost a mark for missing or for incorrect units. Even with an incorrect answer for part (c), candidates did very well on part (d). In part (e) some excellent justifications were given. However, many struggled to convert kilometres to metres, others were confused and compared surface area instead of volume. Some thought the volumes needed to be the same. For part (f) candidates often assumed a right angle at BAW or BWA. When they used the sine and cosine rule, this was mostly done correctly.

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d. Question 4: Trigonometry, volume and area.

   Many were able to write a correct trig ratio for part (a). The most common error was not to write the unrounded or the rounded answer. Some incorrectly used the given value of 186 in their proof. Part (b) was mostly answered correctly, with only a few candidates using Pythagoras' Theorem incorrectly. Most candidates used the correct formula to calculate the volume of the pyramid, but some did not find the correct area for the base of the pyramid. Some lost a mark for missing or for incorrect units. Even with an incorrect answer for part (c), candidates did very well on part (d). In part (e) some excellent justifications were given. However, many struggled to convert kilometres to metres, others were confused and compared surface area instead of volume. Some thought the volumes needed to be the same. For part (f) candidates often assumed a right angle at BAW or BWA. When they used the sine and cosine rule, this was mostly done correctly.

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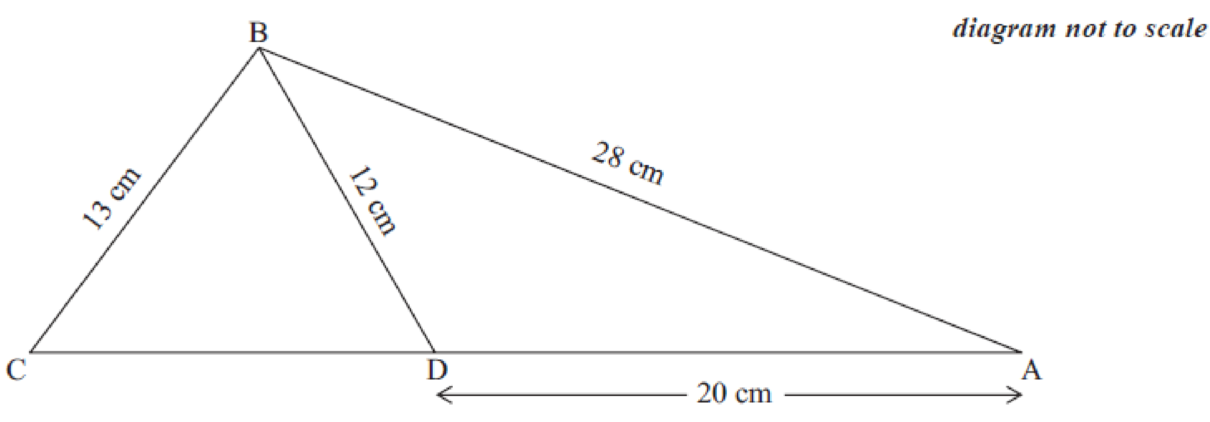
Many were able to write a correct trig ratio for part (a). The most common error was not to write the unrounded or the rounded answer. Some incorrectly used the given value of 186 in their proof. Part (b) was mostly answered correctly, with only a few candidates using Pythagoras' Theorem incorrectly. Most candidates used the correct formula to calculate the volume of the pyramid, but some did not find the correct area for the base of the pyramid. Some lost a mark for missing or for incorrect units. Even with an incorrect answer for part (c), candidates did very well on part (d). In part (e) some excellent justifications were given. However, many struggled to convert kilometres to metres, others were confused and compared surface area instead of volume. Some thought the volumes needed to be the same. For part (f) candidates often assumed a right angle at BAW or BWA. When they used the sine and cosine rule, this was mostly done correctly.

The diagram shows triangle ABC in which $\text{AB} = 28\text{ cm}$, $\text{BC} = 13\text{ cm}$, $\text{BD} = 12\text{ cm}$ and $\text{AD} = 20\text{ cm}$.



a. Calculate the size of angle ADB. [3]

b. Find the area of triangle ADB. [3]

c. Calculate the size of angle BCD. [4]

d. Show that the triangle ABC is not right angled. [4]

Markscheme

a. $\cos \text{ADB} = \frac{12^2+20^2-28^2}{2(12)(20)}$ ***(M1)(A1)***

Notes: Award ***(M1)*** for substituted cosine rule formula, ***(A1)*** for correct substitutions.

\(\angle {\text{ADB}} = 120\) ***(A1)(G2)***

[3 marks]

b. $\text{Area} = \frac{(12)(20)\sin 120^\circ}{2}$ ***(M1)(A1)*****(ft)**

Notes: Award ***(M1)*** for substituted area formula, ***(A1)*****(ft)** for their correct substitutions.

$= 104\ \text{cm}^2\ (103.923\ldots\ \text{cm}^2)$ ***(A1)*****(ft)*****(G2)***

Note: The final answer is $104\ \text{cm}^2$, **the units are required**. Accept $100\ \text{cm}^2$.

[3 marks]

c. $\frac{\sin \text{BCD}}{12} = \frac{\sin 60^\circ}{13}$ ***(A1)*****(ft)*****(M1)(A1)***

Note: Award ***(A1)*****(ft)** for their 60 seen, ***(M1)*** for substituted sine rule formula, ***(A1)*** for correct substitutions.

$\text{BCD} = 53.1^\circ\ (53.0736\ldots)$ ***(A1)(G3)***

Note: Accept 53, do not accept 50 or 53.0.

[4 marks]

d. Using triangle ABC

$\frac{\sin \text{BAC}}{13} = \frac{\sin 53.1^\circ}{28}$ ***(M1)(A1)*****(ft)**

OR

Using triangle ABD

$\frac{\sin \text{BAD}}{12} = \frac{\sin 120^\circ}{28}$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substituted sine rule formula (one of the above), ***(A1)*****(ft)** for their correct substitutions. Follow through from (a) or (c) as appropriate.

$\text{BAC} = \text{BAD} = 21.8^\circ\ (21.7867\ldots)$ ***(A1)*****(ft)*****(G2)***

Notes: Accept 22, do not accept 20 or 21.7. Accept equivalent methods, for example cosine rule.

$180^\circ - (53.1^\circ + 21.8^\circ) \neq 90^\circ$, hence triangle ABC is not right angled ***(R1)(AG)***

OR

$\frac{\text{CD}}{\sin 66.9^\circ} = \frac{13}{\sin 60^\circ}$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substituted sine rule formula, ***(A1)*****(ft)** for their correct substitutions. Follow through from (a) and (c).

$\text{CD} = 13.8\ (13.8075\ldots)$ ***(A1)*****(ft)**

$13^3 + 28^2 \neq 33.8^2$, hence triangle ABC is not right angled. ***(R1)*****(ft)*****(AG)***

Note: The complete statement is required for the final ***(R1)*** to be awarded.

[4 marks]

Examiners report

a. The vast majority of candidates scored very well on this question. Those who did not attempted it using the trigonometry associated with right angled triangles. There were few problems with the use of radians and part (d), which was expected to prove challenging, was successfully overcome by more than half of the candidature. Problems arose mainly because of a lack of clarity in identifying the correct triangle.

b. The vast majority of candidates scored very well on this question. Those who did not attempted it using the trigonometry associated with right angled triangles. There were few problems with the use of radians and part (d), which was expected to prove challenging, was successfully overcome by more than half of the candidature. Problems arose mainly because of a lack of clarity in identifying the correct triangle.

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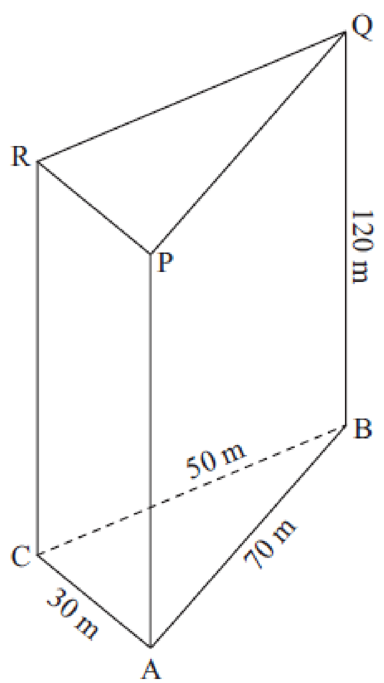
d. The vast majority of candidates scored very well on this question. Those who did not attempted it using the trigonometry associated with right angled triangles. There were few problems with the use of radians and part (d), which was expected to prove challenging, was successfully overcome by more than half of the candidature. Problems arose mainly because of a lack of clarity in identifying the correct triangle.

An office block, ABCPQR, is built in the shape of a triangular prism with its “footprint”, ABC, on horizontal ground. $\text{AB} = 70\text{ m}$, $\text{BC} = 50\text{ m}$ and $\text{AC} = 30\text{ m}$. The vertical height of the office block is 120 m .

diagram not to scale



a. Calculate the size of angle ACB. [3]

b. Calculate the area of the building’s footprint, ABC. [3]

c. Calculate the volume of the office block. [2]

d. To stabilize the structure, a steel beam must be made that runs from point C to point Q. [2]

Calculate the length of CQ.

e. Calculate the angle CQ makes with BC. [2]

Markscheme

a. $\cos \mathrm{ACB} = \frac{30^2+50^2-70^2}{2\times 30\times 50}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted cosine rule formula, ***(A1)*** for correct substitution.

$\mathrm{ACB} = 120^\circ$ ***(A1)(G2)***

b. $\text{Area of triangle ABC} = \frac{30(50)\sin 120^\circ}{2}$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substituted area formula, ***(A1)*****(ft)** for correct substitution.

$= 650\ \mathrm{m}^2\ (649.519\ldots\ \mathrm{m}^2)$ ***(A1)*****(ft)*****(G2)***

Notes: The answer is $650\ \mathrm{m}^2$; the units are required. Follow through from their answer in part (a).

c. $\text{Volume} = 649.519\ldots \times 120$ ***(M1)***

$= 77900\ \mathrm{m}^3\ (77942.2\ldots\ \mathrm{m}^3)$ ***(A1)(G2)***

Note: The answer is $77900\ \mathrm{m}^3$; the units are required. Do not penalise lack of units if already penalized in part (b). Accept $78000\ \mathrm{m}^3$ from use of 3sf answer $650\ \mathrm{m}^2$ from part (b).

d. $\mathrm{CQ}^2 = 50^2 + 120^2$ ***(M1)***

$\mathrm{CQ} = 130\ (\mathrm{m})$ ***(A1)(G2)***

Note: The units are **not** required.

e. $\tan \mathrm{QCB} = \frac{120}{50}$ ***(M1)***

Note: Award ***(M1)*** for correct substituted trig formula.

$\mathrm{QCB} = 67.4^\circ\ (67.3801\ldots)$ ***(A1)(G2)***

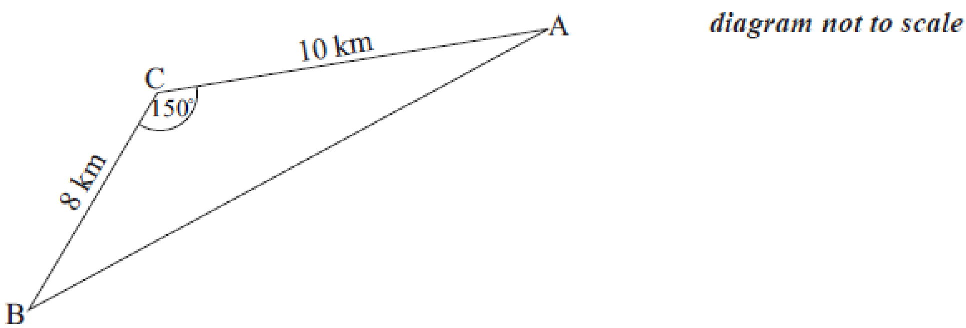
Note: Accept equivalent methods.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

e. [N/A]

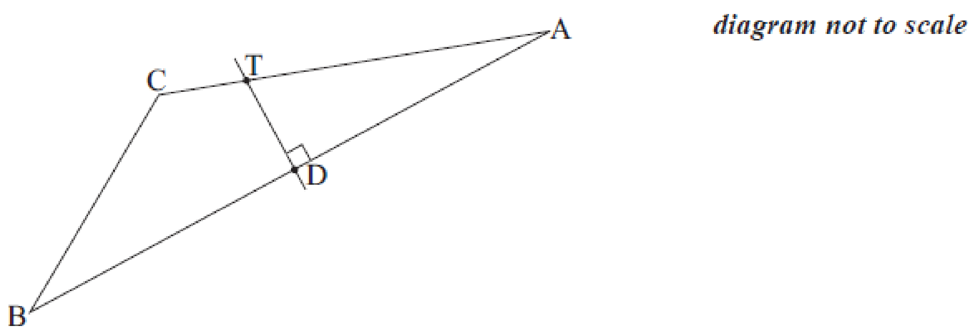
In the diagram below A, B and C represent three villages and the line segments AB, BC and CA represent the roads joining them. The lengths of AC and CB are 10 km and 8 km respectively and the size of the angle between them is 150°.



a. Find the length of the road AB. [3]

b. Find the size of the angle CAB. [3]

c. Village D is halfway between A and B. A new road perpendicular to AB and passing through D is built. Let T be the point where this road cuts AC. This information is shown in the diagram below. [1]



Write down the distance from A to D.

d. Show that the distance from D to T is 2.06 km correct to three significant figures. [2]

e. A bus starts and ends its journey at A taking the route AD to DT to TA. [3]

Find the total distance for this journey.

f. The average speed of the bus while it is moving on the road is 70 km h^{-1}. The bus stops for **5 minutes** at each of D and T . [4]

Estimate the time taken by the bus to complete its journey. Give your answer correct to the nearest minute.

Markscheme

a. $AB^2 = 10^2 + 8^2 - 2 \times 10 \times 8 \times \cos 150°$ ***(M1)(A1)***

AB = 17.4 km ***(A1)(G2)***

Note: Award ***(M1)*** for substitution into correct formula, ***(A1)*** for correct substitution, ***(A1)*** for correct answer.

[3 marks]

b. $\frac{8}{\sin \hat{CAB}} = \frac{17.4}{\sin 150^\circ}$ ***(M1)(A1)***

$\hat{CAB} = 13.3^\circ$ ***(A1)*****(ft)*****(G2)***

Notes: Award ***(M1)*** for substitution into correct formula, ***(A1)*** for correct substitution, ***(A1)*** for correct answer. Follow through from their answer to part (a).

[3 marks]

c. AD = 8.70 km (8.7 km) ***(A1)*****(ft)**

Note: Follow through from their answer to part (a).

[1 mark]

d. DT = tan (13.29...°) × 8.697... = 2.0550... ***(M1)(A1)***

= 2.06 ***(AG)***

Notes: Award ***(M1)*** for correct substitution in the correct formula, award ***(A1)*** for the unrounded answer seen. If 2.06 not seen award at most ***(M1)(AO)***.

[2 marks]

e. $\sqrt{8.70^2 + 2.06^2} + 8.70 + 2.06$ ***(A1)(M1)***

= 19.7 km ***(A1)*****(ft)*****(G2)***

Note: Award ***(A1)*** for AT, ***(M1)*** for adding the three sides of the triangle ADT, ***(A1)*****(ft)** for answer. Follow through from their answer to part (c).

[3 marks]

f. $\frac{19.7}{70} \times 60 + 10$ ***(M1)(M1)***

= 26.9 ***(A1)*****(ft)**

Note: Award ***(M1)*** for time on road in minutes, ***(M1)*** for adding 10, ***(A1)*****(ft)** for unrounded answer. Follow through from their answer to (e).

= 27 (nearest minute) ***(A1)*****(ft)*****(G3)***

Note: Award ***(A1)*****(ft)** for their unrounded answer given to the nearest minute.

[4 marks]

Examiners report

a. The weak students answered parts (a) and (b) using right-angled trigonometry. Different types of mistakes were seen in (a) when applying the cosine rule: some forgot to square root their answer and others calculated each part separately and then missed the 2 minuses. Part (b) was better done than (a). Follow through was applied from (a) to (c). Part (d) was not well done. Most of the students lost one mark in this part question as they did not show the unrounded answer (2.0550...). Part (e) was fairly well done by those who attempted it. In (f) there were very few correct answers. Students found it difficult to find the time when the average speed and distance were given.

b. The weak students answered parts (a) and (b) using right-angled trigonometry. Different types of mistakes were seen in (a) when applying the cosine rule: some forgot to square root their answer and others calculated each part separately and then missed the 2 minuses. Part (b) was better done than (a). Follow through was applied from (a) to (c). Part (d) was not well done. Most of the students lost one mark in this part question as they did not show the unrounded answer (2.0550...). Part (e) was fairly well done by those who attempted it. In (f) there were very few correct answers. Students found it difficult to find the time when the average speed and distance were given.

c. The weak students answered parts (a) and (b) using right-angled trigonometry. Different types of mistakes were seen in (a) when applying the cosine rule: some forgot to square root their answer and others calculated each part separately and then missed the 2 minuses. Part (b) was better done than (a). Follow through was applied from (a) to (c). Part (d) was not well done. Most of the students lost one mark in this part question as they did not show the unrounded answer (2.0550...). Part (e) was fairly well done by those who attempted it. In (f) there were very few correct answers. Students found it difficult to find the time when the average speed and distance were given.

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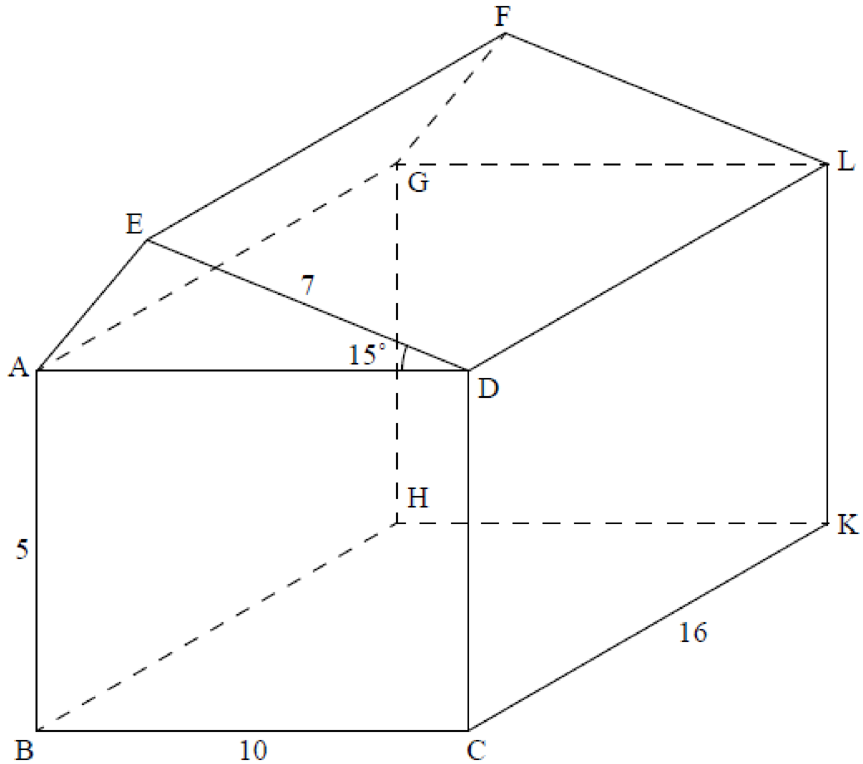
Farmer Brown has built a new barn, on horizontal ground, on his farm. The barn has a cuboid base and a triangular prism roof, as shown in the diagram.

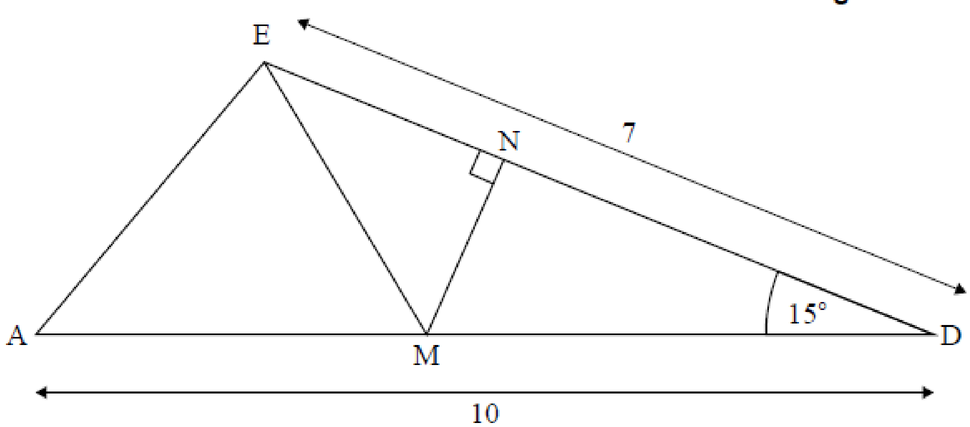
diagram not to scale



The cuboid has a width of 10 m, a length of 16 m and a height of 5 m.
The roof has two sloping faces and two vertical and identical sides, ADE and GLF.
The face DEFL slopes at an angle of 15° to the horizontal and ED = 7 m .

The roof was built using metal supports. Each support is made from **five** lengths of metal AE, ED, AD, EM and MN, and the design is shown in the following diagram.

diagram not to scale



ED = 7 m , AD = 10 m and angle ADE = 15° .
M is the midpoint of AD.
N is the point on ED such that MN is at right angles to ED.

Farmer Brown believes that N is the midpoint of ED.

a. Calculate the area of triangle EAD. [3]

b. Calculate the **total** volume of the barn. [3]

c. Calculate the length of MN. [2]

d. Calculate the length of AE. [3]

e. Show that Farmer Brown is incorrect. [3]

f. Calculate the **total** length of metal required for one support. [4]

Markscheme

a. (Area of EAD =) $\frac{1}{2} \times 10 \times 7 \times \sin 15$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into area of a triangle formula, **(A1)** for correct substitution. Award ***(M0)(A0)(A0)*** if EAD or AED is considered to be a right-angled triangle.

= 9.06 m^2 (9.05866… m^2) ***(A1) (G3)***

[3 marks]

b. (10 × 5 × 16) + (9.05866… × 16) ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitution into volume of a cuboid, ***(M1)*** for adding the correctly substituted volume of their triangular prism.

= 945 m^3 (944.938… m^3) ***(A1)*****(ft)** ***(G3)***

Note: Follow through from part (a).

[3 marks]

c. $\frac{\text{MN}}{5} = \sin 15$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into trigonometric equation.

(MN =) 1.29(m) (1.29409… (m)) ***(A1) (G2)***

[2 marks]

d. (AE^2 =) $10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 15$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into cosine rule formula, and ***(A1)*** for correct substitution.

(AE =) 3.71(m) (3.71084… (m)) ***(A1) (G2)***

[3 marks]

e. $ND^2 = 5^2 - (1.29409\ldots)^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras theorem.

(ND =) 4.83 (4.82962…) ***(A1)*****(ft)**

Note: Follow through from part (c).

OR

$\frac{1.29409\ldots}{\text{ND}} = \tan 15^\circ$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into tangent.

(ND =) 4.83 (4.82962…) ***(A1)*****(ft)**

Note: Follow through from part (c).

OR

$\frac{\text{ND}}{5} = \cos 15^\circ$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into cosine.

(ND =) 4.83 (4.82962…) ***(A1)*(ft)**

Note: Follow through from part (c).

OR

$ND^2 = 1.29409…^2 + 5^2 − 2 × 1.29409… × 5 × \cos 75°$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into cosine rule.

(ND =) 4.83 (4.82962…) ***(A1)*(ft)**

Note: Follow through from part (c).

4.82962… ≠ 3.5 (ND ≠ 3.5) ***(R1)*(ft)**

OR

4.82962… ≠ 2.17038… (ND ≠ NE) ***(R1)*(ft)**

(hence Farmer Brown is incorrect)

Note: Do not award ***(M0)(A0)(R1)*(ft)**. Award ***(M0)(A0)(R0)*** for a correct conclusion without any working seen.

[3 marks]

f. $(EM^2 =) 1.29409…^2 + (7 − 4.82962…)^2$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into Pythagoras theorem.

OR

$(EM^2 =) 5^2 + 7^2 − 2 × 5 × 7 × \cos 15$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into cosine rule formula.

(EM =) 2.53(m) (2.52689...(m)) ***(A1)*(ft) *(G2)*(ft)**

Note: Follow through from parts (c), (d) and (e).

(Total length =) 2.52689… + 3.71084… + 1.29409… +10 + 7 ***(M1)***

Note: Award ***(M1)*** for adding their EM, their parts (c) and (d), and 10 and 7.

= 24.5 (m) (24.5318… (m)) ***(A1)*(ft) *(G4)***

Note: Follow through from parts (c) and (d).

[4 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

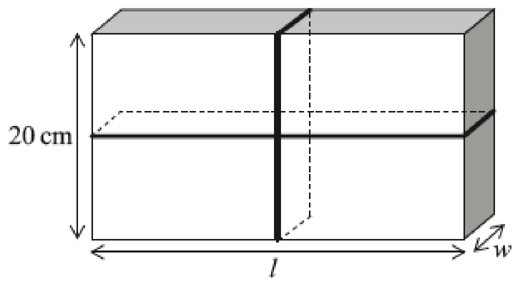
A parcel is in the shape of a rectangular prism, as shown in the diagram. It has a length l cm, width w cm and height of 20 cm.

The total volume of the parcel is $3000\ \text{cm}^3$.

a. Express the volume of the parcel in terms of l and w. [1]

b. Show that $l = \frac{150}{w}$. [2]

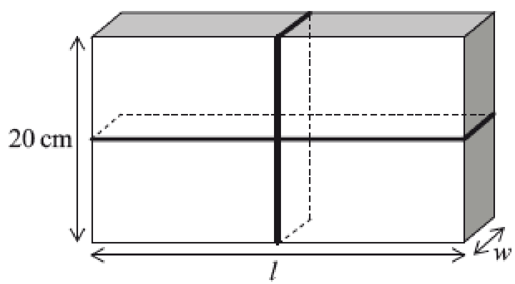
c. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



Show that the length of string, S cm, required to tie up the parcel can be written as

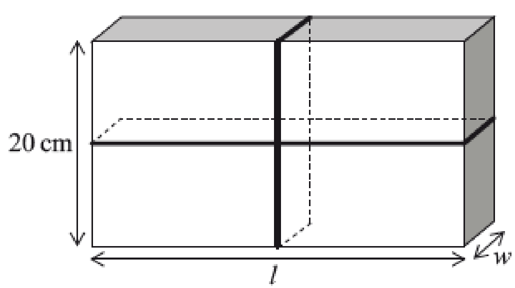
$$S = 40 + 4w + \frac{300}{w},\ 0 < w \leqslant 20.$$

d. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



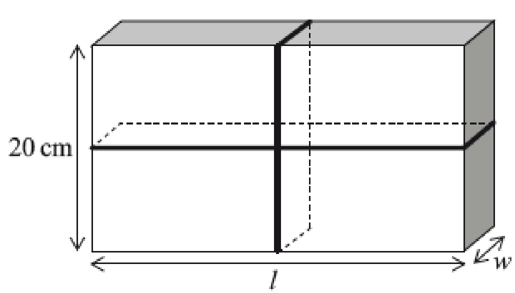
Draw the graph of S for $0 < w \leqslant 20$ and $0 < S \leqslant 500$, clearly showing the local minimum point. Use a scale of 2 cm to represent 5 units on the horizontal axis w (cm), and a scale of 2 cm to represent 100 units on the vertical axis S (cm).

e. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [3]



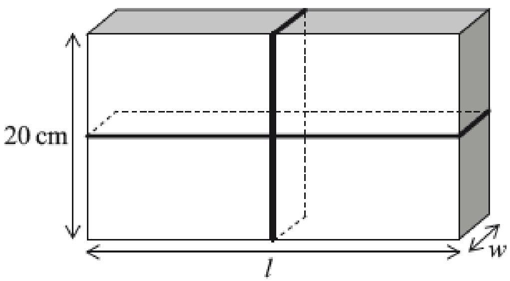
Find $\frac{\mathrm{d}S}{\mathrm{d}w}$.

f. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



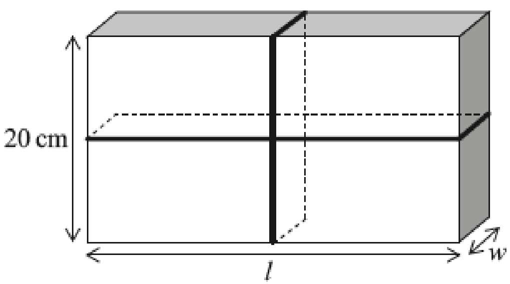
Find the value of w for which S is a minimum.

g. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [1]



Write down the value, l, of the parcel for which the length of string is a minimum.

h. The parcel is tied up using a length of string that fits **exactly** around the parcel, as shown in the following diagram. [2]



Find the minimum length of string required to tie up the parcel.

Markscheme

a. $20lw$ **OR** $V = 20lw$ ***(A1)***

[1 mark]

b. $3000 = 20lw$ ***(M1)***

Note: Award ***(M1)*** for equating their answer to part (a) to 3000.

$l = \frac{3000}{20w}$ ***(M1)***

Note: Award ***(M1)*** for rearranging equation to make l subject of the formula. The above equation must be seen to award ***(M1)***.

OR

$150 = lw$ ***(M1)***

Note: Award ***(M1)*** for division by 20 on both sides. The above equation must be seen to award ***(M1)***.

$l = \frac{150}{w}$ ***(AG)***

[2 marks]

c. $S = 2l + 4w + 2(20)$ ***(M1)***

Note: Award ***(M1)*** for setting up a correct expression for S.

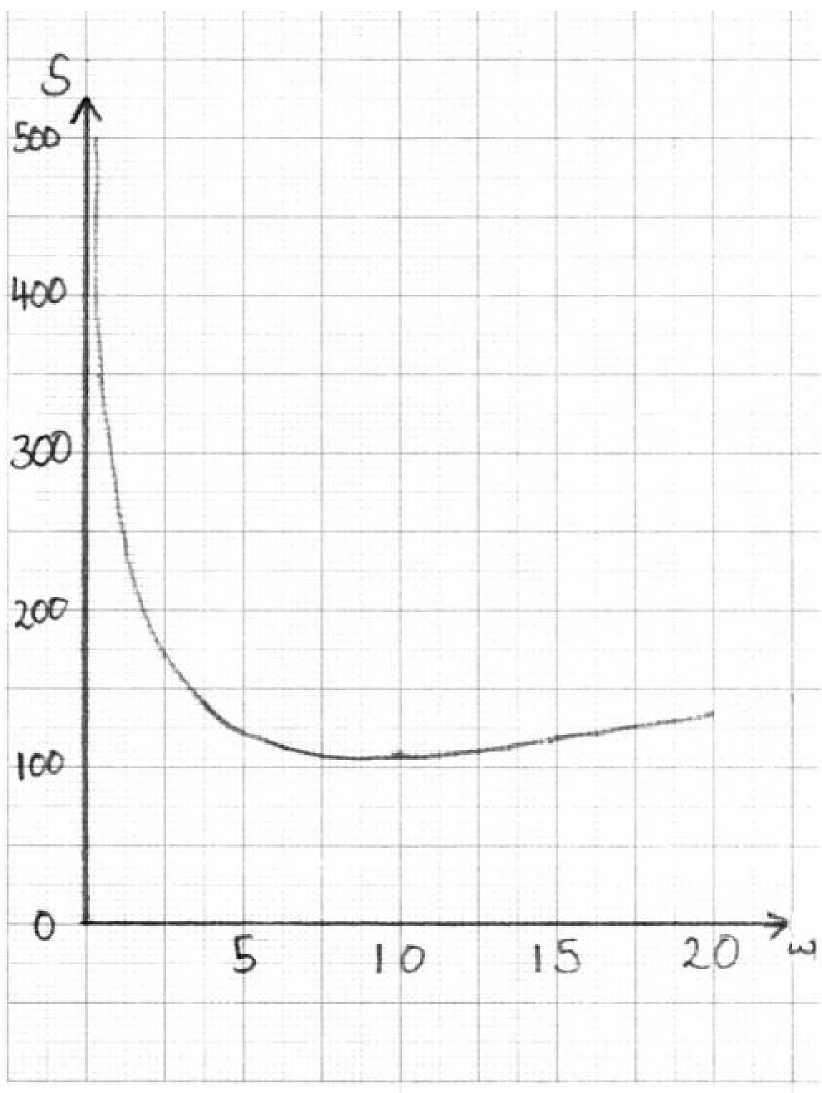
$2\left(\frac{150}{w}\right)+4w+2(20)$ ***(M1)***

Notes: Award ***(M1)*** for correct substitution into the expression for S. The above expression must be seen to award ***(M1)***.

$=40+4w+\frac{300}{w}$ ***(AG)***

[2 marks]

d.



(A1)(A1)(A1)(A1)

Note: Award ***(A1)*** for correct scales, window and labels on axes, ***(A1)*** for approximately correct shape, ***(A1)*** for minimum point in approximately correct position, ***(A1)*** for asymptotic behaviour at $w=0$.

Axes must be drawn with a ruler and labeled w and S.

For a smooth curve (with approximately correct shape) there should be **one** continuous thin line, no part of which is straight and no (one-to-many) mappings of w.

The S-axis must be an asymptote. The curve must not touch the S-axis nor must the curve approach the asymptote then deviate away later.

[4 marks]

e. $4 - \frac{300}{w^2}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for 4, ***(A1)*** for -300, ***(A1)*** for $\frac{1}{w^2}$ or w^{-2}. If extra terms present, award at most ***(A1)(A1)(A0)***.

[3 marks]

f. $4 - \frac{300}{w^2} = 0$ **OR** $\frac{300}{w^2} = 4$ **OR** $\frac{\mathrm{d}S}{\mathrm{d}w} = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their derivative to zero.

$w = 8.66 \left(\sqrt{75},\ 8.66025\ldots\right)$ ***(A1)*(ft)*(G2)***

Note: Follow through from their answer to part (e).

[2 marks]

g. $17.3\left(\frac{150}{\sqrt{75}},\ 17.3205\ldots\right)$ ***(A1)*(ft)**

Note: Follow through from their answer to part (f).

[1 mark]

h. $40 + 4\sqrt{75} + \frac{300}{\sqrt{75}}$ ***(M1)***

Note: Award ***(M1)*** for substitution of their answer to part (f) into the expression for S.

$= 110\ (\mathrm{cm})\ \left(40 + 40\sqrt{3},\ 109.282\ldots\right)$ ***(A1)*(ft)*(G2)***

Note: Do not accept 109.

Follow through from their answers to parts (f) and (g).

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]
[N/A]

h.

ABC is a triangular field on horizontal ground. The lengths of AB and AC are 70 m and 50 m respectively. The size of angle BCA is 78°.

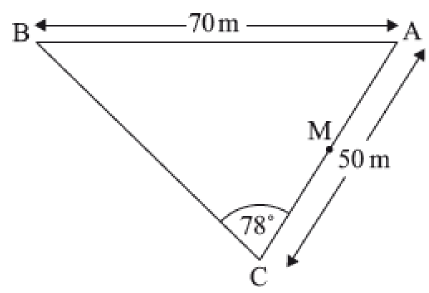


diagram not to scale

a. Find the size of angle ABC. [3]

b. Find the area of the triangular field. [4]

c. M is the midpoint of AC. [3]

Find the length of BM.

d. A vertical mobile phone mast, TB, is built next to the field with its base at B. The angle of elevation of T from M is 63.4°. N is the midpoint of the mast. [5]

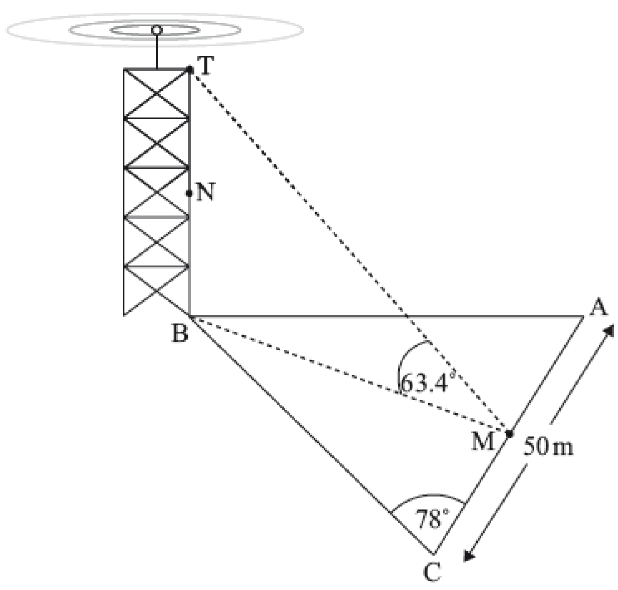


diagram not to scale

Calculate the angle of elevation of N from M.

Markscheme

a. $\frac{70}{\sin 78} = \frac{50}{\sin \mathrm{A\hat{B}C}}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted sine rule, ***(A1)*** for correct substitution.

$\hat{ABC} = 44.3°$ (44.3209...) ***(A1)(G3)***

Note: If radians are used the answer is $0.375918...$, award at most ***(M1)(A1)(A0)***.

[3 marks]

b. area $\Delta ABC = \frac{1}{2} \times 70 \times 50 \times \sin(57.6790\ldots)$ ***(A1)(M1)(A1)*(ft)**

Notes: Award ***(A1)*(ft)** for their $57.6790\ldots$ seen, ***(M1)*** for substituted area formula, ***(A1)*(ft)** for correct substitution.

Follow through from part (a).

$= 1480 \text{ m}^2 \ \ (1478.86\ldots)$ ***(A1)*(ft)*(G3)***

Notes: The answer is 1480 m^2, units are required. $1479.20\ldots$ if 3 sf used.

If radians are used the answer is $1554.11\ldots \text{m}^2$, award ***(A1)*(ft)*(M1)(A1)*(ft)*(A1)*(ft)*(G3)***.

[4 marks]

c. $BM^2 = 70^2 + 25^2 - 2 \times 70 \times 25 \times \cos(57.6790\ldots)$ ***(M1)(A1)(ft)***

Notes: Award ***(M1)*** for substituted cosine rule, ***(A1)*(ft)** for correct substitution. Follow through from their angle in part (b).

$BM = 60.4 \text{ (m)} \ \ (60.4457\ldots)$ ***(A1)*(ft)*(G2)***

Notes: If the 3 sf answer is used the answer is 60.5 (m).

If radians are used the answer is $62.5757\ldots \text{ (m)}$, award ***(M1)(A1)*(ft)*(A1)*(ft)*(G2)***.

[3 marks]

d. $\tan 63.4° = \frac{TB}{60.4457\ldots}$ ***(M1)***

Note: Award ***(M1)*** for their correctly substituted trig equation.

$TB = 120.707\ldots$ ***(A1)*(ft)**

Notes: Follow through from part (c). If 3 sf answers are used throughout, $TB = 120.815\ldots$

If $TB = 120.707\ldots$ is seen without working, award ***(A2)***.

$\tan N\hat{M}B = \frac{\left(\frac{120.707\ldots}{2}\right)}{60.4457\ldots}$ ***(A1)*(ft)*(M1)***

Notes: Award ***(A1)*(ft)** for their TB divided by 2 seen, ***(M1)*** for their correctly substituted trig equation.

Follow through from part (c) and **within part (d)**.

$\mathrm{N\hat{M}B} = 45.0^\circ \quad (44.9563\ldots)$ ***(A1)*(ft)*(G3)***

Notes: If 3 sf are used throughout, answer is 45°.

If radians are used the answer is $0.308958\ldots$, and if full working is shown, award at most ***(M1)(A1)*(ft)*(A1)*(ft)*(M1)(A0)***.

If no working is shown for radians answer, award ***(G2)***.

OR

$\tan \mathrm{N\hat{M}B} = \frac{\mathrm{NB}}{\mathrm{BM}}$ ***(M1)***

$\tan 63.4^\circ = \frac{2\times\mathrm{NB}}{\mathrm{BM}}$ ***(A1)(M1)***

Note: Award ***(A1)*** for $2 \times \mathrm{NB}$ seen.

$\tan \mathrm{N\hat{M}B} = \frac{1}{2}\tan 63.4^\circ$ ***(M1)***

$\mathrm{N\hat{M}B} = 45.0^\circ \quad (44.9563\ldots)$ ***(A1)(G3)***

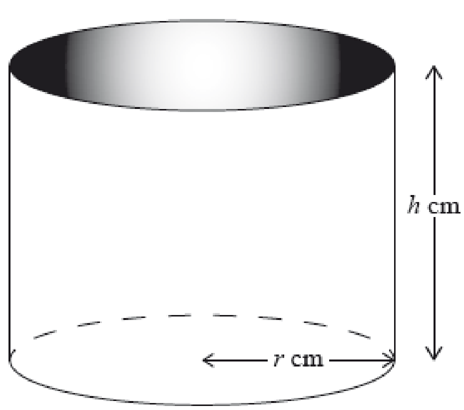
Notes: If radians are used the answer is $0.308958\ldots$, and if full working is shown, award at most ***(M1)(A1)(M1)(M1)(A0)***. If no working is shown for radians answer, award ***(G2)***.

[5 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

A water container is made in the shape of a cylinder with internal height h cm and internal base radius r cm.



The water container has no top. The inner surfaces of the container are to be coated with a water-resistant material.

The volume of the water container is $0.5\ \mathrm{m}^3$.

The water container is designed so that the area to be coated is minimized.

One can of water-resistant material coats a surface area of $2000\ \mathrm{cm}^2$.

a. Write down a formula for A, the surface area to be coated. [2]

b. Express this volume in cm^3. [1]

c. Write down, in terms of r and h, an equation for the volume of this water container. [1]

d. Show that $A = \pi r^2 \frac{1\,000\,000}{r}$. [2]

d. Show that $A = \pi r^2 + \frac{1\,000\,000}{r}$. [2]

e. Find $\frac{\mathrm{d}A}{\mathrm{d}r}$. [3]

f. Using your answer to part (e), find the value of r which minimizes A. [3]

g. Find the value of this minimum area. [2]

h. Find the least number of cans of water-resistant material that will coat the area in part (g). [3]

Markscheme

a. $(A =)\ \pi r^2 + 2\pi rh$ ***(A1)(A1)***

Note: Award ***(A1)*** for either πr^2 **OR** $2\pi rh$ seen. Award ***(A1)*** for two correct terms added together.

[2 marks]

b. $500\,000$ ***(A1)***

Notes: Units **not** required.

[1 mark]

c. $500\,000 = \pi r^2 h$ ***(A1)*****(ft)**

Notes: Award ***(A1)*****(ft)** for $\pi r^2 h$ equating to their part (b).

Do not accept unless $V = \pi r^2 h$ is explicitly defined as their part (b).

[1 mark]

d. $A=\pi r^2+2\pi r\left(\frac{500\,000}{\pi r^2}\right)$ ***(A1)*****(ft)*****(M1)***

Note: Award ***(A1)*****(ft)** for their $\frac{500\,000}{\pi r^2}$ seen.

Award ***(M1)*** for correctly substituting **only** $\frac{500\,000}{\pi r^2}$ into a **correct** part (a).

Award ***(A1)*****(ft)*****(M1)*** for rearranging part (c) to $\pi rh=\frac{500\,000}{r}$ and substituting for πrh in expression for A.

$A=\pi r^2+\frac{1\,000\,000}{r}$ ***(AG)***

Notes: The conclusion, $A=\pi r^2+\frac{1\,000\,000}{r}$, must be consistent with their working seen for the ***(A1)*** to be awarded.

Accept 10^6 as equivalent to $1\,000\,000$.

[2 marks]

d. $A=\pi r^2+2\pi r\left(\frac{500\,000}{\pi r^2}\right)$ ***(A1)*****(ft)*****(M1)***

Note: Award ***(A1)*****(ft)** for their $\frac{500\,000}{\pi r^2}$ seen.

Award ***(M1)*** for correctly substituting **only** $\frac{500\,000}{\pi r^2}$ into a **correct** part (a).

Award ***(A1)*****(ft)*****(M1)*** for rearranging part (c) to $\pi rh=\frac{500\,000}{r}$ and substituting for πrh in expression for A.

$A=\pi r^2+\frac{1\,000\,000}{r}$ ***(AG)***

Notes: The conclusion, $A=\pi r^2+\frac{1\,000\,000}{r}$, must be consistent with their working seen for the ***(A1)*** to be awarded.

Accept 10^6 as equivalent to $1\,000\,000$.

[2 marks]

e. $2\pi r-\frac{1\,000\,000}{r^2}$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for $2\pi r$, ***(A1)*** for $\frac{1}{r^2}$ or r^{-2}, ***(A1)*** for $-1\,000\,000$.

[3 marks]

f. $2\pi r-\frac{1\,000\,000}{r^2}=0$ ***(M1)***

Note: Award ***(M1)*** for equating their part (e) to zero.

$r^3=\frac{1\,000\,000}{2\pi}$ **OR** $r=\sqrt[3]{\frac{1\,000\,000}{2\pi}}$ ***(M1)***

Note: Award ***(M1)*** for isolating r.

OR

sketch of derivative function ***(M1)***

with its zero indicated ***(M1)***

$(r =)\ 54.2\ (\text{cm})\ (54.1926\ldots)$ ***(A1)*(ft)*(G2)***

[3 marks]

g. $\pi(54.1926\ldots)^2 + \frac{1\,000\,000}{(54.1926\ldots)}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their part (f) into the given equation.

$= 27\,700\ (\text{cm}^2)\ (27\,679.0\ldots)$ ***(A1)*(ft)*(G2)***

[2 marks]

h. $\frac{27\,679.0\ldots}{2000}$ ***(M1)***

Note: Award ***(M1)*** for dividing their part (g) by 2000.

$= 13.8395\ldots$ ***(A1)*(ft)**

Notes: Follow through from part (g).

14 (cans) ***(A1)*(ft)*(G3)***

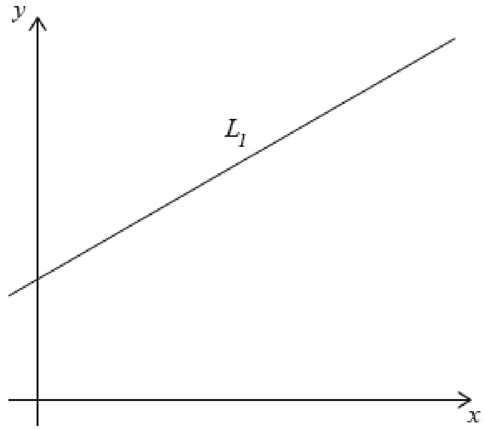
Notes: Final ***(A1)*** awarded for rounding up their $13.8395\ldots$ to the next integer.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]
h. [N/A]

The line L_1 has equation $2y - x - 7 = 0$ and is shown on the diagram.



The point A has coordinates $(1,\ 4)$.

The point C has coordinates $(5,\ 12)$. M is the midpoint of AC.

The straight line, L_2, is perpendicular to AC and passes through M.

The point D is the intersection of L_1 and L_2.

The length of MD is $\frac{\sqrt{45}}{2}$.

The point B is such that ABCD is a rhombus.

a. Show that A lies on L_1. [2]

b. Find the coordinates of M. [2]

c. Find the length of AC. [2]

d. Show that the equation of L_2 is $2y + x - 19 = 0$. [5]

e. Find the coordinates of D. [2]

f. Write down the length of MD correct to five significant figures. [1]

g. Find the area of ABCD. [3]

Markscheme

a. $2 \times 4 - 1 - 7 = 0$ (or equivalent) ***(R1)***

Note: For ***(R1)*** accept substitution of $x = 1$ or $y = 4$ into the equation followed by a confirmation that $y = 4$ or $x = 1$.

(since the point satisfies the equation of the line,) A lies on L_1 ***(A1)***

Note: Do not award ***(A1)(R0)***.

[2 marks]

b. $\frac{1+5}{2}$ **OR** $\frac{4+12}{2}$ seen ***(M1)***

Note: Award ***(M1)*** for at least one correct substitution into the midpoint formula.

$(3,\ 8)$ ***(A1)(G2)***

Notes: Accept $x = 3,\ y = 8$.

Award ***(M1)(A0)*** for $\left(\frac{1+5}{2},\ \frac{4+12}{2}\right)$.

Award ***(G1)*** for each correct coordinate seen without working.

[2 marks]

c. $\sqrt{(5-1)^2 + (12-4)^2}$ ***(M1)***

Note: Award ***(M1)*** for a correct substitution into distance between two points formula.

$= 8.94\ \left(4\sqrt{5},\ \sqrt{80},\ 8.94427\ldots\right)$ ***(A1)(G2)***

[2 marks]

d. gradient of $\mathrm{AC} = \frac{12-4}{5-1}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into gradient formula.

$= 2$ ***(A1)***

Note: Award ***(M1)(A1)*** for gradient of $\mathrm{AC} = 2$ with or without working

gradient of the normal $= -\frac{1}{2}$ ***(M1)***

Note: Award ***(M1)*** for the negative reciprocal of their gradient of AC.

$y - 8 = -\frac{1}{2}(x-3)$ **OR** $8 = -\frac{1}{2}(3) + c$ ***(M1)***

Note: Award ***(M1)*** for substitution of their point and gradient into straight line formula. This ***(M1)*** can **only** be awarded where $-\frac{1}{2}$ (gradient) is correctly determined as the gradient of the normal to AC.

$2y - 16 = -(x-3)$ **OR** $-2y + 16 = x - 3$ **OR** $2y = -x + 19$ ***(A1)***

Note: Award ***(A1)*** for correctly removing fractions, **but only** if their equation is equivalent to the given equation.

$2y + x - 19 = 0$ ***(AG)***

Note: The conclusion $2y + x - 19 = 0$ must be seen for the ***(A1)*** to be awarded.

Where the candidate has **shown** the gradient of the normal to $\mathrm{AC} = -0.5$, award ***(M1)*** for $2(8) + 3 - 19 = 0$ and ***(A1)*** for (therefore) $2y + x - 19 = 0$.

Simply substituting $(3,\ 8)$ into the equation of L_2 with no other prior working, earns no marks.

[5 marks]

e. $(6,\ 6.5)$ ***(A1)(A1)(G2)***

Note: Award ***(A1)*** for 6, ***(A1)*** for 6.5. Award a maximum of ***(A1)(A0)*** if answers are not given as a coordinate pair. Accept $x = 6,\ y = 6.5$.

Award ***(M1)(A0)*** for an attempt to solve the two simultaneous equations $2y - x - 7 = 0$ and $2y + x - 19 = 0$ algebraically, leading to at least one incorrect or missing coordinate.

[2 marks]

f. 3.3541 ***(A1)***

Note: Answer must be to 5 significant figures.

[1 mark]

g. $2 \times \frac{1}{2} \times \sqrt{80} \times \frac{\sqrt{45}}{2}$ ***(M1)(M1)***

Notes: Award ***(M1)*** for correct substitution into area of triangle formula.

If their triangle is a quarter of the rhombus then award ***(M1)*** for multiplying their triangle by 4.

If their triangle is a half of the rhombus then award ***(M1)*** for multiplying their triangle by 2.

OR

$\frac{1}{2} \times \sqrt{80} \times \sqrt{45}$ ***(M1)(M1)***

Notes: Award ***(M1)*** for doubling MD to get the diagonal BD, ***(M1)*** for correct substitution into the area of a rhombus formula.

Award ***(M1)(M1)*** for $\sqrt{80}\times$ their (f).

$= 30$ ***(A1)*****(ft)*****(G3)***

Notes: Follow through from parts (c) and (f).

$8.94 \times 3.3541 = 29.9856\ldots$

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]

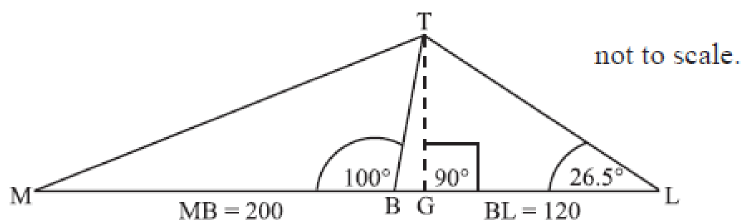
Jenny has a circular cylinder with a lid. The cylinder has height 39 **cm** and diameter 65 **mm**.

An old tower (BT) leans at 10° away from the vertical (represented by line TG).

The base of the tower is at B so that $\mathrm{M\hat{B}T} = 100^\circ$.

Leonardo stands at L on flat ground 120 m away from B in the direction of the lean.

He measures the angle between the ground and the top of the tower T to be $\mathrm{B\hat{L}T} = 26.5^\circ$.



i.a. Calculate the volume of the cylinder **in cm^3**. Give your answer correct to **two** decimal places. [3]

i.b. The cylinder is used for storing tennis balls. Each ball has a **radius** of 3.25 cm. [1]

Calculate how many balls Jenny can fit in the cylinder if it is filled to the top.

i.c. (i) Jenny fills the cylinder with the number of balls found in part (b) and puts the lid on. Calculate the volume of air inside the cylinder in the spaces between the tennis balls. [4]

(ii) Convert your answer to (c) (i) into cubic metres.

ii.a. (i) Find the value of angle $\mathrm{B\hat{T}L}$. [5]

(ii) Use triangle BTL to calculate the sloping distance BT from the base, B to the top, T of the tower.

ii.b. Calculate the vertical height TG of the top of the tower. [2]

ii.c. Leonardo now walks to point M, a distance 200 m from B on the opposite side of the tower. Calculate the distance from M to the top of the tower at T. [3]

Markscheme

i.a. $\pi \times 3.25^2 \times 39$ ***(M1)(A1)***

(= 1294.1398)

Answer 1294.14 (cm^3)(2dp) ***(A1)*(ft)*(G2)***

(UP) *not applicable in this part due to wording of question.* ***(M1)*** *is for substituting appropriate numbers from the problem into the correct formula, even if the units are mixed up.* ***(A1)*** *is for correct substitutions or correct answer with more than 2dp in cubic centimetres seen. Award* ***(G1)*** *for answer to > 2dp with no working and no attempt to correct to 2dp. Award* ***(M1)(A0)(A1)(ft)*** *for* $\pi \times 32.5^2 \times 39 \text{ cm}^3$ (= 129413.9824) = 129413.98

Use of $\pi = \frac{22}{7}$ **or** *3.142 etc is premature rounding and is awarded at most* ***(M1)(A1)(A0)*** *or* ***(M1)(A0)(A1)(ft)*** *depending on whether the intermediate value is seen or not. For all other incorrect substitutions, award* ***(M1)(A0)*** *and only follow through the 2 dp correction if the intermediate answer to more decimal places is seen. Answer given as a multiple of* π *is awarded at most* ***(M1)(A1)(A0).*** *As usual, an* ***unsubstituted*** *formula followed by correct answer only receives the G marks.*

[3 marks]

i.b. 39/6.5 = 6 ***(A1)***

[1 mark]

i.c. *Unit penalty* ***(UP)*** *is applicable where indicated in the left hand column.*

(UP) (i) Volume of one ball is $\frac{4}{3}\pi \times 3.25^3 \text{ cm}^3$ ***(M1)***

$\text{Volume of air} = \pi \times 3.25^2 \times 39 - 6 \times \frac{4}{3}\pi \times 3.25^3 = 431 \text{ cm}^3$ ***(M1)(A1)*(ft)*(G2)***

Award first ***(M1)*** *for substituted volume of sphere formula or for numerical value of sphere volume seen (143.79… or 45.77…* $\times\pi$*). Award second* ***(M1)*** *for subtracting candidate's sphere volume multiplied by their answer to (b). Follow through from parts (a) and (b) only, but negative or zero answer is always awarded* ***(A0)*(ft)**

(UP) (ii) 0.000431m^3 or 4.31×10^{-4} m^3 ***(A1)*(ft)**

[4 marks]

ii.a. *Unit penalty* ***(UP)*** *is applicable where indicated in the left hand column.*

(i) $\text{Angle B}\widehat{\text{T}}\text{L} = 180 - 80 - 26.5$ or $180 - 90 - 26.5 - 10$ ***(M1)***

$= 73.5^\circ$ ***(A1)(G2)***

(ii) $\frac{BT}{\sin(26.5^\circ)} = \frac{120}{\sin(73.5^\circ)}$ ***(M1)(A1)*(ft)**

(UP) BT = 55.8 m (3sf) ***(A1)*(ft)**

[5 marks]

If radian mode has been used throughout the question, award ***(A0)*** *to the first incorrect answer then follow through, but negative lengths are always awarded* ***(A0)(ft).***

The answers are (all 3sf)

(ii)(a) – *124 m* ***(A0)*(ft)**

(ii)(b) *123 m* ***(A0)***

(ii)(c) *313 m* ***(A0)***

If radian mode has been used throughout the question, award ***(A0)*** *to the first incorrect answer then follow through, but negative lengths are always awarded* ***(A0)*(ft)** ii.b. *Unit penalty **(UP)** is applicable where indicated in the left hand column.*

TG = 55.8sin(80°) or 55.8cos(10°) ***(M1)***

(UP) = 55.0 m (3sf) ***(A1)*****(ft)*****(G2)***

*Apply **(AP)** if 0 missing*

[2 marks]

*If radian mode has been used throughout the question, award **(A0)** to the first incorrect answer then follow through, but negative lengths are always awarded **(A0)(ft)**.*

The answers are (all 3sf)

*(ii)(a) – 124 m **(A0)***(ft)

*(ii)(b) 123 m **(A0)***

*(ii)(c) 313 m **(A0)***

*If radian mode has been used throughout the question, award **(A0)** to the first incorrect answer then follow through, but negative lengths are always awarded **(A0)***(ft)

ii.c. *Unit penalty **(UP)** is applicable where indicated in the left hand column.*

$\mathrm{MT}^2 = 200^2 + 55.8^2 - 2 \times 200 \times 55.8 \times \cos(100°)$ ***(M1)(A1)*****(ft)**

(UP) MT = 217 m (3sf) ***(A1)*****(ft)**

Follow through only from part (ii)(a)(ii). Award marks at discretion for any valid alternative method.

[3 marks]

*If radian mode has been used throughout the question, award **(A0)** to the first incorrect answer then follow through, but negative lengths are always awarded **(A0)(ft)**.*

The answers are (all 3sf)

*(ii)(a) – 124 m **(A0)***(ft)

*(ii)(b) 123 m **(A0)***

*(ii)(c) 313 m **(A0)***

*If radian mode has been used throughout the question, award **(A0)** to the first incorrect answer then follow through, but negative lengths are always awarded **(A0)***(ft)

Examiners report

i.a. (i) Many candidates incurred the new one-off unit penalty here. Too many ignored the call for two decimal places and some extrapolated that instruction to later parts (which was clearly not intended). There was the predictable confusion of using radius instead of diameter. Another common error was to divide the cylinder volume by that of the ball, to decide how many would fit. Some follow-through was allowed later from this error, however, this led to zero or negligible air volume, which was clearly ridiculous.

Choice and use of the formulae for volumes was often competent but the conversion to cubic metres was very badly done. Almost no correct answers were seen at all.

i.b. (i) Many candidates incurred the new one-off unit penalty here. Too many ignored the call for two decimal places and some extrapolated that instruction to later parts (which was clearly not intended). There was the predictable confusion of using radius instead of diameter. Another common error was to divide the cylinder volume by that of the ball, to decide how many would fit. Some follow-through was allowed later from this error, however, this led to zero or negligible air volume, which was clearly ridiculous.

Choice and use of the formulae for volumes was often competent but the conversion to cubic metres was very badly done. Almost no correct answers were seen at all.

i.c. (i) Many candidates incurred the new one-off unit penalty here. Too many ignored the call for two decimal places and some extrapolated that instruction to later parts (which was clearly not intended). There was the predictable confusion of using radius instead of diameter. Another common error was to divide the cylinder volume by that of the ball, to decide how many would fit. Some follow-through was allowed later from this error, however, this led to zero or negligible air volume, which was clearly ridiculous.

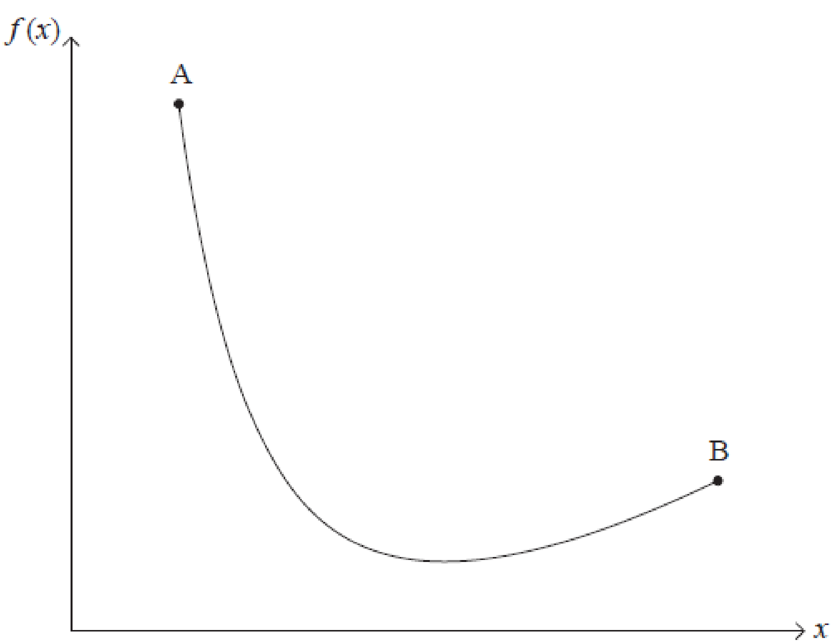
Choice and use of the formulae for volumes was often competent but the conversion to cubic metres was very badly done. Almost no correct answers were seen at all.

ii.a. (ii) Candidates were often sloppy in reading the information. In particular, despite the statement BL = 120 clearly written, many took GL as 120. Triangle TBL was often taken as right-angled. Angle BTL presented few problems, though sometimes the method was very long-winded. Candidates often managed part (a) then went awry in later parts. Many unit penalties were applied, if not already used in questions 1 or 2.

ii.b. (ii) Candidates were often sloppy in reading the information. In particular, despite the statement BL = 120 clearly written, many took GL as 120. Triangle TBL was often taken as right-angled. Angle BTL presented few problems, though sometimes the method was very long-winded. Candidates often managed part (a) then went awry in later parts. Many unit penalties were applied, if not already used in questions 1 or 2.

ii.c. (ii) Candidates were often sloppy in reading the information. In particular, despite the statement BL = 120 clearly written, many took GL as 120. Triangle TBL was often taken as right-angled. Angle BTL presented few problems, though sometimes the method was very long-winded. Candidates often managed part (a) then went awry in later parts. Many unit penalties were applied, if not already used in questions 1 or 2.

The graph of the function $f(x) = \frac{14}{x} + x - 6$, for $1 \leq x \leq 7$ is given below.



a. Calculate $f(1)$. [2]

b. Find $f'(x)$. [3]

c. **Use your answer to part (b)** to show that the *x*-coordinate of the local minimum point of the graph of f is 3.7 correct to 2 significant figures. [3]

d. Find the range of f. [3]

e. Points A and B lie on the graph of f. The *x*-coordinates of A and B are 1 and 7 respectively. [1]

Write down the *y*-coordinate of B.

f. Points A and B lie on the graph of f . The *x*-coordinates of A and B are 1 and 7 respectively. [2]

Find the gradient of the straight line passing through A and B.

g. M is the midpoint of the line segment AB. [2]

Write down the coordinates of M.

h. *L* is the tangent to the graph of the function $y = f(x)$, at the point on the graph with the same *x*-coordinate as M. [2]

Find the gradient of *L*.

i. Find the equation of *L*. Give your answer in the form $y = mx + c$. [3]

Markscheme

a. $\frac{14}{(1)} + (1) - 6$ ***(M1)***

Note: Award ***(M1)*** for substituting $x = 1$ into f.

$= 9$ ***(A1)(G2)***

b. $-\frac{14}{x^2} + 1$ ***(A3)***

Note: Award ***(A1)*** for -14, ***(A1)*** for $\frac{14}{x^2}$ or for x^{-2}, ***(A1)*** for 1.

Award at most ***(A2)*** if any extra terms are present.

c. $-\frac{14}{x^2} + 1 = 0$ or $f'(x) = 0$ ***(M1)***

Note: Award ***(M1)*** for equating **their** derivative in part (b) to 0.

$\frac{14}{x^2} = 1$ or $x^2 = 14$ or equivalent ***(M1)***

Note: Award ***(M1)*** for correct rearrangement of their equation.

$x = 3.74165...(\sqrt{14})$ ***(A1)***

$x = 3.7$ ***(AG)***

Notes: Both the unrounded and rounded answers must be seen to award the ***(A1)***. This is a “show that” question; appeals to their GDC are not accepted –award a maximum of ***(M1)(M0)(A0)***.

Specifically, $-\frac{14}{x^2} + 1 = 0$ followed by $x = 3.74165...$, $x = 3.7$ is awarded ***(M1)(M0)(A0)***.

d. $1.48 \leqslant y \leqslant 9$ ***(A1)(A1)*(ft)*(A1)***

Note: Accept alternative notations, for example [1.48,9]. ($x = \sqrt{14}$ leads to answer 1.48331...)

Note: Award ***(A1)*** for 1.48331…seen, accept 1.48378… from using the given answer $x = 3.7$, ***(A1)*(ft)** for their 9 from part (a) seen, ***(A1)*** for the correct notation for their interval (accept $\leqslant y \leqslant$ or $\leqslant f \leqslant$).

e. 3 ***(A1)***

Note: Do not accept a coordinate pair.

f. $\frac{3-9}{7-1}$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into the gradient formula.

$= -1$ ***(A1)*(ft)*(G2)***

Note: Follow through from their answers to parts (a) and (e).

g. (4, 6) ***(A1)*(ft)*(A1)***

Note: Accept $x = 4$, $y = 6$. Award at most ***(A1)(A0)*** if parentheses not seen.

If coordinates reversed award ***(A0)(A1)*(ft)**.

Follow through from their answers to parts (a) and (e).

h. $-\frac{14}{4^2} + 1$ ***(M1)***

Note: Award ***(M1)*** for substitution into their gradient function.

Follow through from their answers to parts (b) and (g).

$= \frac{1}{8}(0.125)$ ***(A1)*(ft)*(G2)***

i. $y - 1.5 = \frac{1}{8}(x - 4)$ ***(M1)*(ft)*(M1)***

Note: Award ***(M1)*** for substituting their (4, 1.5) in any straight line formula,

(M1) for substituting their gradient in any straight line formula.

$y = \frac{x}{8} + 4$ ***(A1)*(ft)*(G2)***

Note: The form of the line has been specified in the question.

Examiners report

a. Most candidates were able to evaluate the function and find the derivative for $x + 6$ but the term with the negative index was problematic. The few candidates who equated their derivative to zero at the local minimum point progressed well and showed a thorough understanding of the differential calculus. Many did not attain full marks for the range of the function, either confusing this with the statistical concept of range or using the y-coordinate at B. Most were able to find the gradient and midpoint of the straight line passing through A and B. The final parts were also challenging for the majority: many had difficulty finding the gradient of the tangent L, instead using the slope formula for a straight line; the most common error in part (i) was to substitute in the coordinates of midpoint M rather than the point on the curve. Greater insight into the problem would have come from using the given sketch of the curve and annotating it; it seems that many candidates do not link the algebraic nature of the differential calculus with the curve in question.

b. Most candidates were able to evaluate the function and find the derivative for $x + 6$ but the term with the negative index was problematic. The few candidates who equated their derivative to zero at the local minimum point progressed well and showed a thorough understanding of the differential calculus. Many did not attain full marks for the range of the function, either confusing this with the statistical concept of range or using the y-coordinate at B. Most were able to find the gradient and midpoint of the straight line passing through A and B. The final parts were also challenging for the majority: many had difficulty finding the gradient of the tangent L, instead using the slope formula for a straight line; the most common error in part (i) was to substitute in the coordinates of midpoint M rather than the point on the curve. Greater insight into the problem would have come from using the given sketch of the curve and annotating it; it seems that many candidates do not link the algebraic nature of the differential calculus with the curve in question.

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e. Most candidates were able to evaluate the function and find the derivative for $x + 6$ but the term with the negative index was problematic. The few candidates who equated their derivative to zero at the local minimum point progressed well and showed a thorough understanding of the differential calculus. Many did not attain full marks for the range of the function, either confusing this with the statistical concept of range or using the y-coordinate at B. Most were able to find the gradient and midpoint of the straight line passing through A and B. The final parts were also challenging for the majority: many had difficulty finding the gradient of the tangent L, instead using the slope formula for a straight line; the most common error in part (i) was to substitute in the coordinates of midpoint M rather than the point on the curve. Greater insight into the problem would have come from using the given sketch of the curve and annotating it; it seems that many candidates do not link the algebraic nature of the differential calculus with the curve in question.

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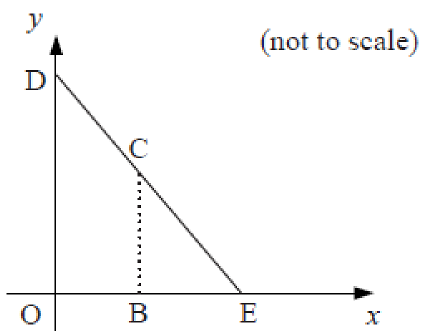
On the coordinate axes below, D is a point on the y-axis and E is a point on the x-axis. O is the origin. The equation of the line DE is $y + \frac{1}{2}x = 4$

.



a. Write down the coordinates of point E. [2]

b. C is a point on the line DE. B is a point on the x-axis such that BC is parallel to the y-axis. The x-coordinate of C is t. [2]

Show that the y-coordinate of C is $4 - \frac{1}{2}t$.

c. OBCD is a trapezium. The y-coordinate of point D is 4. [3]

Show that the area of OBCD is $4t - \frac{1}{4}t^2$.

d. The area of OBCD is 9.75 square units. Write down a quadratic equation that expresses this information. [1]

e. (i) Using your graphic display calculator, or otherwise, find the two solutions to the quadratic equation written in part (d). [4]

(ii) Hence find the correct value for t. Give a reason for your answer.

Markscheme

a. $\mathrm{E}(8, 0)$ ***(A1)(A1)***

Notes: Brackets required but do not penalize again if mark lost in **Q4** (i)(d). If missing award ***(A1)(A0)***.
Accept $x = 8$, $y = 0$
Award ***(A1)*** for $x = 8$

b. $y + \frac{1}{2}t = 4$ ***(M1)(M1)***

Note: ***(M1)*** for the equation of the line seen. ***(M1)*** for substituting t.

$y = 4 - \frac{1}{2}t$ ***(AG)***

Note: Final line must be seen or previous ***(M1)*** mark is lost.

[2 marks]

c. $\text{Area} = \frac{1}{2} \times (4 + 4 - \frac{1}{2}t) \times t$ ***(M1)(A1)***

Note: ***(M1)*** for substituting in correct formula, ***(A1)*** for correct substitution.

$= \frac{1}{2} \times (8 - \frac{1}{2}t) \times t = \frac{1}{2}(8t - \frac{1}{2}t^2)$ ***(A1)***
$= 4t - \frac{1}{4}t^2$ ***(AG)***

Note: Final line must be seen or previous ***(A1)*** mark is lost.

[3 marks]

d. $4t - \frac{1}{4}t^2 = 9.75$ or any equivalent form. ***(A1)***

[1 mark]

e. (i) $t = 3$ or $t = 13$ ***(A1)*(ft)*(A1)*(ft)*(G2)***

Note: Follow through from candidate's equation to part (d). Award ***(A0)(A1)*(ft)** for $(3, 0)$ and $(13, 0)$.

(ii) t must be a value between 0 and 8 then $t = 3$

Note: Accept B is between O and E. Do not award ***(R0)(A1)***.

Examiners report

a. A number of candidates did not attempt this question worth 12 marks but the majority answered this question partially and were able to gain some marks. Parts (a) and (b) were mostly well done. Very few candidates managed to answer part (c) well; this part of the question required good algebra along with a clear understanding of the situation given in the diagram. Many recovered then in (d) when they were asked to write down the quadratic equation. Solving the equation was not always found to be easy. Use of the GDC was expected but many used the formula. The correct solution, $t = 3$, was chosen in the last part of the question. However, their justification was often false causing them to lose both the reasoning and the answer mark.

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A dog food manufacturer has to cut production costs. She wishes to use as little aluminium as possible in the construction of cylindrical cans. In the following diagram, h represents the height of the can in cm and x, the radius of the base of the can in cm.

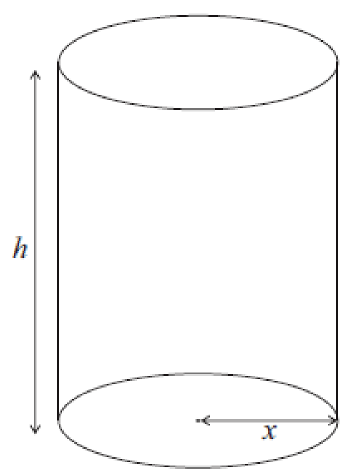

diagram not to scale

The volume of the dog food cans is 600 cm^3.

a. Show that $h = \frac{600}{\pi x^2}$. [2]

b.i. Find an expression for the curved surface area of the can, in terms of x. Simplify your answer. [2]

b.ii. Hence write down an expression for A, the total surface area of the can, in terms of x. [2]

c. Differentiate *A* in terms of *x*. [3]

d. Find the value of *x* that makes *A* a minimum. [3]

e. Calculate the minimum total surface area of the dog food can. [2]

Markscheme

a. $600 = \pi x^2 h$ ***(M1)(A1)***

$\frac{600}{\pi x^2} = h$ ***(AG)***

Note: Award ***(M1)*** for correct substituted formula, ***(A1)*** for correct substitution. If answer given not shown award at most ***(M1)(A0)***.

[2 marks]

b.i. $C = 2\pi x \frac{600}{\pi x^2}$ ***(M1)***

$C = \frac{1200}{x}$ (or 1200x^{-1}) ***(A1)***

Note: Award ***(M1)*** for correct substitution in formula, ***(A1)*** for correct simplification.

[??? marks]

b.ii. $A = 2\pi x^2 + 1200x^{-1}$ ***(A1)(A1)*****(ft)**

Note: Award ***(A1)*** for multiplying the area of the base by two, ***(A1)*** for adding on their answer to part (b) (i).

For both marks to be awarded answer must be in terms of *x*.

[??? marks]

c. $\frac{\mathrm{d}A}{\mathrm{d}x} = 4\pi x - \frac{1200}{x^2}$ ***(A1)*****(ft)*****(A1)*****(ft)*****(A1)*****(ft)**

Notes: Award ***(A1)*** for $4\pi x$, ***(A1)*** for -1200, ***(A1)*** for x^{-2}. Award at most ***(A2)*** if any extra term is written. Follow through from their part (b) (ii).

[??? marks]

d. $4\pi x - \frac{1200}{x^2} = 0$ ***(M1)(M1)***

$x^3 = \frac{1200}{4\pi}$ (or equivalent)

$x = 4.57$ ***(A1)*****(ft)*****(G2)***

Note: Award ***(M1)*** for using their derivative, ***(M1)*** for setting the derivative to zero, ***(A1)*****(ft)** for answer.

Follow through from their derivative.

Last mark is lost if value of *x* is zero or negative.

[3 marks]

e. $A = 2\pi(4.57)^2 + 1200(4.57)^{-1}$ ***(M1)***

$A = 394$ ***(A1)*(ft)*(G2)***

Note: Follow through from their answers to parts (b) (ii) and (d).

[2 marks]

Examiners report

a. This was the most difficult question for the candidates. It was clear that the vast majority of them had not had exposure to this style of question. Part (a) was well answered by most of the students. In part (b) the correct expression “in terms of x ” for the curve surface area was not frequently seen. In many cases the impression was that they did not know what “in terms of x ” meant as correct equivalent expressions were seen but where the h was also involved. Those candidates that made progress in the question, even with the wrong expression for the total area of the can, A were able to earn follow through marks.

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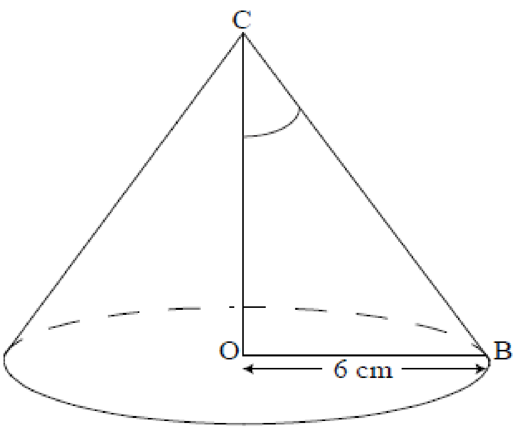
A solid metal **cylinder** has a base radius of 4 cm and a height of 8 cm.

a. Find the area of the base of the cylinder. [2]

b. Show that the volume of the metal used in the cylinder is 402 cm^3, given correct to three significant figures. [2]

c. Find the total surface area of the cylinder. [3]

d. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. [3]



Find the height, OC, of the cone.

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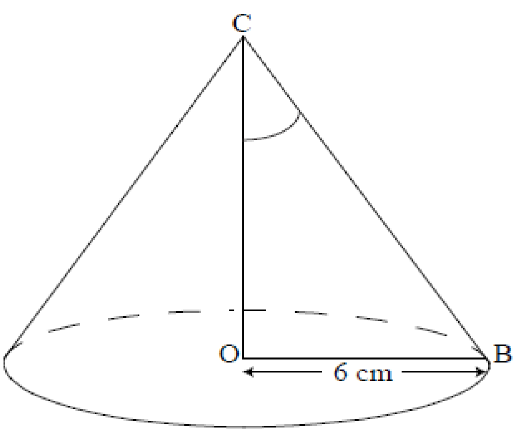
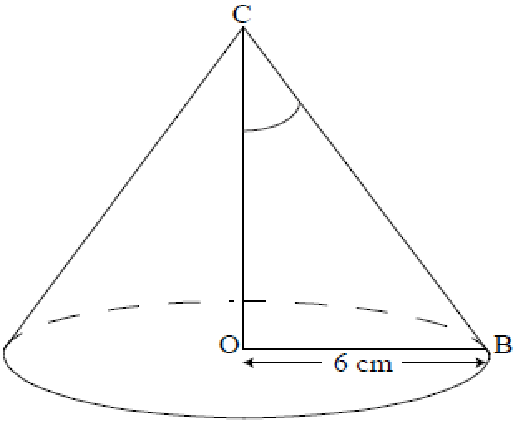


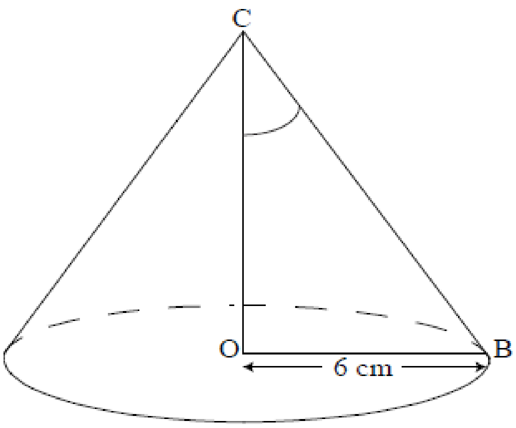
Find the size of angle BCO.

f. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. [2]



Find the slant height, CB.

g. The cylinder was melted and recast into a solid cone, shown in the following diagram. The base radius OB is 6 cm. [4]



Find the total surface area of the cone.

Markscheme

a. $\pi \times 4^2$ ***(M1)***

= 50.3 (16π) cm^2 (50.2654...) ***(A1)(G2)***

Note: Award ***(M1)*** for correct substitution in area formula. The answer is 50.3 cm^2, the units are required.

[2 marks]

b. 50.265...× 8 ***(M1)***

Note: Award ***(M1)*** for correct substitution in the volume formula.

= 402.123... ***(A1)***
= 402 (cm^3) ***(AG)***

Note: Both the unrounded and the rounded answer must be seen for the ***(A1)*** to be awarded. The units are **not** required

[2 marks]

c. $2 \times \pi \times 4 \times 8 + 2 \times \pi \times 4^2$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitution in the curved surface area formula, ***(M1)*** for adding the area of their two bases.

= 302 cm^2 (96π cm^2) (301.592...) ***(A1)*(ft)*(G2)***

Notes: The answer is 302 cm^2, the units are required. Do not penalise for missing or incorrect units if penalised in part (a). Follow through from their answer to part (a).

[3 marks]

d. $\frac{1}{3}\pi \times 6^2 \times \text{OC} = 402$ ***(M1)(M1)***

Note: Award ***(M1)*** for correctly substituted volume formula, ***(M1)*** for equating to 402 (402.123…).

$\text{OC} = 10.7\ (\text{cm})\ \left(10\frac{2}{3},\ 10.6666...\right)$ ***(A1)(G2)***

[3 marks]

e. $\tan \text{BCO} = \frac{6}{10.66...}$ ***(M1)***

Note: Award ***(M1)*** for use of correct tangent ratio.

$\text{B}\hat{\text{C}}\text{O} = 29.4^\circ$ (29.3577...) ***(A1)*(ft)*(G2)***

Notes: Accept 29.3° (29.2814...) if 10.7 is used. An acceptable alternative method is to calculate CB first and then angle BCO. Allow follow through from parts (d) and (f). Answers range from 29.2° to 29.5°.

[2 marks]

f. $\text{CB} = \sqrt{6^2 + (10.66...)^2}$ ***(M1)***

OR

$\sin 29.35...^\circ = \frac{6}{\text{CB}}$ ***(M1)***

OR

$\cos 29.35...^\circ = \frac{10.66...}{\text{CB}}$ ***(M1)***

CB = 12.2 (cm) (12.2383...) ***(A1)*(ft)*(G2)***

Note: Accept 12.3 (12.2674...) if 10.7 (and/or 29.3) used. Follow through from part (d) or part (e) as appropriate.

[2 marks]

g. $\pi \times 6 \times 12.2383... + \pi \times 6^2$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for correct substitution in curved surface area formula, ***(M1)*** for correct substitution in area of circle formula, ***(M1)*** for addition of the two areas.

= 344 cm^2 (343.785...) ***(A1)*(ft)*(G3)***

Note: The answer is 344 cm^2, the units are required. Do not penalise for missing or incorrect units if already penalised in either part (a) or (c). Accept 345 cm^2 if 12.3 is used and 343 cm^2 if 12.2 is used. Follow through from their part (f).

[4 marks]

Examiners report

a. This question was either very well done – by the majority – or very poorly (but not both). Many incomplete attempts were seen. This would perhaps indicate a lack of preparation in this area of the syllabus from some centres, since it was that the formulas for cones were not well understood. Further, the idea of “total surface area” was a mystery to many – a slavish reliance of formulas, irrespective of context, led to many errors and a consequent loss of marks.

The invariance of volume for solids and liquids that provided the link in this question was not understood by many, but was felt to be an appropriate subject for an examination.

b. This question was either very well done – by the majority – or very poorly (but not both). Many incomplete attempts were seen. This would perhaps indicate a lack of preparation in this area of the syllabus from some centres, since it was that the formulas for cones were not well understood. Further, the idea of “total surface area” was a mystery to many – a slavish reliance of formulas, irrespective of context, led to many errors and a consequent loss of marks.

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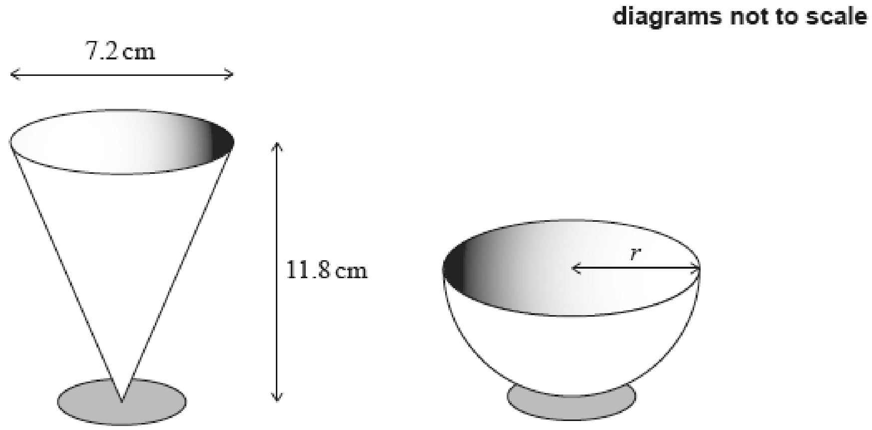
   The invariance of volume for solids and liquids that provided the link in this question was not understood by many, but was felt to be an appropriate subject for an examination.

A restaurant serves desserts in glasses in the shape of a cone and in the shape of a hemisphere. The diameter of a cone shaped glass is 7.2 cm and the height of the cone is 11.8 cm as shown.

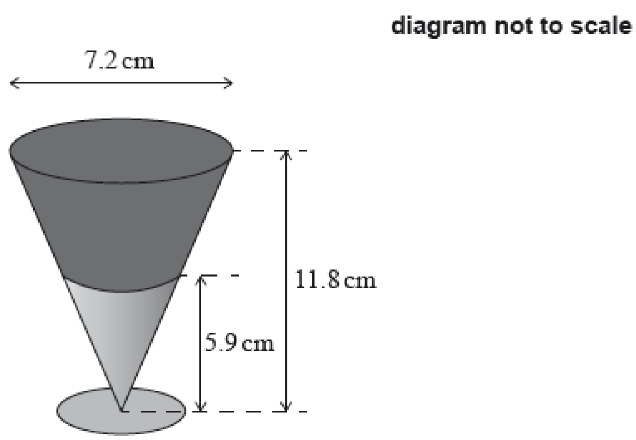


The volume of a hemisphere shaped glass is $225\ \text{cm}^3$.

The restaurant offers two types of dessert.

The **regular dessert** is a hemisphere shaped glass completely filled with chocolate mousse. The cost, to the restaurant, of the chocolate mousse for one regular dessert is $1.89.

The **special dessert** is a cone shaped glass filled with two ingredients. It is first filled with orange paste to half of its height and then with chocolate mousse for the remaining volume.



The cost, to the restaurant, of $100\ \text{cm}^3$ of orange paste is $7.42.

A chef at the restaurant prepares 50 desserts; x regular desserts and y special desserts. The cost of the ingredients for the 50 desserts is $111.44.

a. Show that the volume of a cone shaped glass is $160\ \text{cm}^3$, correct to 3 significant figures. [2]

b. Calculate the radius, r, of a hemisphere shaped glass. [3]

c. Find the cost of $100\ \mathrm{cm}^3$ of chocolate mousse. [2]

d. Show that there is $20\ \mathrm{cm}^3$ of orange paste in each special dessert. [2]

e. Find the total cost of the ingredients of one special dessert. [2]

f. Find the value of x. [3]

Markscheme

a. $(V =)\ \frac{1}{3}\pi(3.6)^2 \times 11.8$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into volume of a cone formula.

$= 160.145\ldots\ (\mathrm{cm}^3)$ ***(A1)***

$= 160\ (\mathrm{cm}^3)$ ***(AG)***

Note: Both rounded and unrounded answers must be seen for the final ***(A1)*** to be awarded.

[2 marks]

b. $\frac{1}{2} \times \frac{4}{3}\pi r^3 = 225$ ***(M1)(A1)***

Notes: Award ***(M1)*** for multiplying volume of sphere formula by $\frac{1}{2}$ (or equivalent).

Award ***(A1)*** for equating the volume of hemisphere formula to 225.

OR

$\frac{4}{3}\pi r^3 = 450$ ***(A1)(M1)***

Notes: Award ***(A1)*** for 450 seen, ***(M1)*** for equating the volume of sphere formula to 450.

$(r =)\ 4.75\ (\mathrm{cm})\ (4.75380\ldots)$ ***(A1)(G2)***

[3 marks]

c. $\frac{1.89 \times 100}{225}$ ***(M1)***

Note: Award ***(M1)*** for dividing 1.89 by 2.25, or equivalent.

$= 0.84$ ***(A1)(G2)***

Note: Accept 84 cents if the units are explicit.

[2 marks]

d. $r_2 = 1.8$ ***(A1)***

$V_2 = \frac{1}{3}\pi(1.8)^2 \times 5.9$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into volume of a cone formula, but only if the result rounds to 20.

$= 20\ \text{cm}^3$ ***(AG)***

OR

$r_2 = \frac{1}{2}r$ ***(A1)***

$V_2 = \left(\frac{1}{2}\right)^3 160$ ***(M1)***

Notes: Award ***(M1)*** for multiplying 160 by $\left(\frac{1}{2}\right)^3$. Award ***(A0)(M1)*** for $\frac{1}{8} \times 160$ if $\frac{1}{2}$ is not seen.

$= 20\ (\text{cm}^3)$ ***(AG)***

Notes: Do not award any marks if the response substitutes in the known value $(V = 20)$ to find the radius of the cone.

[2 marks]

e. $\frac{20}{100} \times 7.42 + \frac{140}{100} \times 0.84$ ***(M1)***

Note: Award ***(M1)*** for the sum of two correct products.

$ 2.66 ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (c).

[2 marks]

f. $x + y = 50$ ***(M1)***

Note: Award ***(M1)*** for correct equation.

$1.89x + 2.66y = 111.44$ ***(M1)***

Note: Award ***(M1)*** for setting up correct equation, including their 2.66 from part (e).

$(x =)\ 28$ ***(A1)*****(ft)*****(G3)***

Note: Follow through from part (e), but only if their answer for x is rounded to the nearest positive integer, where $0 < x < 50$.

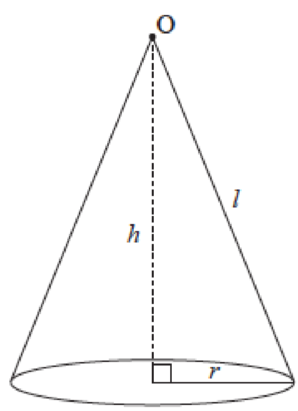
Award at most ***(M1)(M1)(A0)*** for a final answer of “28, 22”, where the x-value is not clearly defined.

[3 marks]

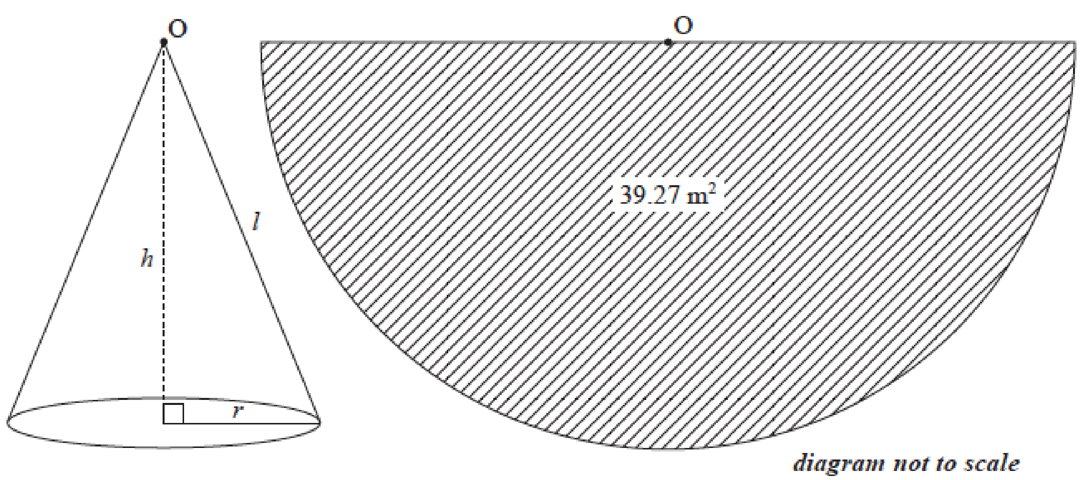
Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

Tepees were traditionally used by nomadic tribes who lived on the Great Plains of North America. They are cone-shaped dwellings and can be modelled as a cone, with vertex O, shown below. The cone has radius, r, height, h, and slant height, l.



A model tepee is displayed at a Great Plains exhibition. The curved surface area of this tepee is covered by a piece of canvas that is 39.27 m^2, and has the shape of a semicircle, as shown in the following diagram.



a. Show that the slant height, l, is 5 m, correct to the nearest metre. [2]

b. (i) Find the circumference of the base of the cone. [6]

(ii) Find the radius, r, of the base.

(iii) Find the height, h.

c. A company designs cone-shaped tents to resemble the traditional tepees. [1]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Write down an expression for the height, h, in terms of the radius, r, of these cone-shaped tents.

d. A company designs cone-shaped tents to resemble the traditional tepees. [1]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Show that the volume of the tent, V, can be written as

$$V = 3.11\pi r^2 - \frac{2}{3}\pi r^3.$$

e. A company designs cone-shaped tents to resemble the traditional tepees. [2]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

Find $\frac{\mathrm{d}V}{\mathrm{d}r}$.

f. A company designs cone-shaped tents to resemble the traditional tepees. [4]

These cone-shaped tents come in a range of sizes such that the sum of the diameter and the height is equal to **9.33 m**.

(i) Determine the exact value of r for which the volume is a maximum.

(ii) Find the maximum volume.

Markscheme

a. $\frac{\pi l^2}{2} = 39.27$ ***(M1)(A1)***

Note: Award ***(M1)*** for equating the formula for area of a semicircle to 39.27, award ***(A1)*** for correct substitution of l into the formula for area of a semicircle.

$l = 5\ (\mathrm{m})$ ***(AG)***

b. (i) $5 \times \pi$ ***(M1)***

$= 15.7 \quad (15.7079..., \ 5\pi)\ (\mathrm{m})$ ***(A1)(G2)***

(ii) $2\pi r = 15.7079\ldots$ **OR** $5\pi r = 39.27$ ***(M1)***

$(r =)\ 2.5\ (\mathrm{m})$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (b)(i).

(iii) $(h^2 =)\ 5^2 - 2.5^2$ ***(M1)***

Notes: Award ***(M1)*** for correct substitution into Pythagoras' theorem. Follow through from part (b)(ii).

$(h =)\ 4.33\ (4.33012\ldots)\ (\mathrm{m})$ ***(A1)*(ft)*(G2)***

c. $9.33 - 2 \times r$ ***(A1)***

d. $V = \frac{\pi r^2}{3} \times (9.33 - 2r)$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the volume formula.

$V = 3.11\pi r^2 - \frac{2}{3}\pi^3$ ***(AG)***

e. $6.22\pi r - 2\pi r^2$ ***(A1)(A1)***

Notes: Award ***(A1)*** for $6.22\pi r$, ***(A1)*** for $-2\pi r^2$.

If extra terms present, award at most ***(A1)(A0)***.

f. (i) $6.22\pi r - 2\pi r^2 = 0$ ***(M1)***

Note: Award ***(M1)*** for setting their derivative from part (e) to 0.

$r = 3.11\ (\mathrm{m})$ ***(A1)*(ft)*(G2)***

Notes: Award ***(A1)*** for identifying 3.11 as the answer.

Follow through from their answer to part (e).

(ii) $\frac{1}{3}\pi(3.11)^3$ **OR** $3.11\pi(3.11)^2 - \frac{2}{3}\pi(3.11)^3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the correct volume formula.

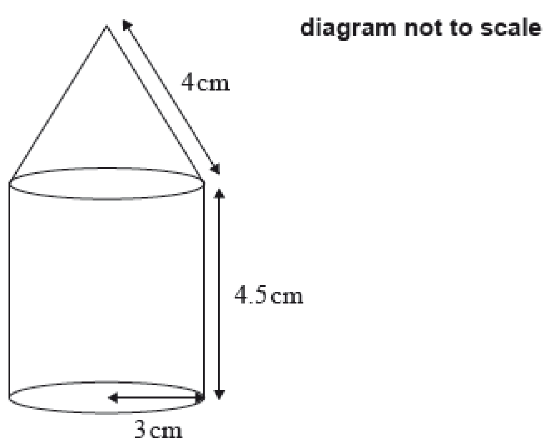
$31.5\ (\mathrm{m}^3)(31.4999\ldots)$ ***(A1)*(ft)*(G2)***

Note: Follow through from their answer to part (f)(i).

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

The following diagram shows a perfume bottle made up of a cylinder and a cone.



The radius of both the cylinder and the base of the cone is 3 cm.

The height of the cylinder is 4.5 cm.

The slant height of the cone is 4 cm.

a. (i) Show that the vertical height of the cone is 2.65 cm correct to three significant figures. [6]

(ii) Calculate the volume of the perfume bottle.

b. The bottle contains 125 cm^3 of perfume. The bottle is **not** full and all of the perfume is in the cylinder part. [2]

Find the height of the perfume in the bottle.

c. Temi makes some crafts with perfume bottles, like the one above, once they are empty. Temi wants to know the surface area of one perfume bottle. [4]

Find the **total** surface area of the perfume bottle.

d. Temi covers the perfume bottles with a paint that costs 3 South African rand (ZAR) per millilitre. One millilitre of this paint covers an area of 7 cm^2. [4]

Calculate the cost, in ZAR, of painting the perfume bottle. **Give your answer correct to two decimal places**.

e. Temi sells her perfume bottles in a craft fair for 325 ZAR each. Dominique from France buys one and wants to know how much she has spent, in euros (EUR). The exchange rate is 1 EUR = 13.03 ZAR. [2]

Find the price, in EUR, that Dominique paid for the perfume bottle. **Give your answer correct to two decimal places**.

Markscheme

a. (i) $x^2 + 3^2 = 4^2$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras' formula.

Accept correct alternative method using trigonometric ratios.

$x = 2.64575\ldots$ ***(A1)***

$x = 2.65 \text{ (cm)}$ ***(AG)***

Note: The unrounded and rounded answer must be seen for the ***(A1)*** to be awarded.

OR

$\sqrt{4^2 - 3^2}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into Pythagoras' formula.

$= \sqrt{7}$ ***(A1)***

$= 2.65 \text{ (cm)}$ ***(AG)***

Note: The exact answer must be seen for the final ***(A1)*** to be awarded.

(ii) $\pi \times 3^2 \times 4.5 + \frac{1}{3}\pi \times 3^2 \times 2.65$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for correct substitution into the volume of a cylinder formula, ***(M1)*** for correct substitution into the volume of a cone formula, ***(M1)*** for adding both of their volumes.

$= 152 \text{ cm}^3 \quad (152.210\ldots \text{ cm}^3,\ 48.45\pi \text{ cm}^3)$ ***(A1)(G3)***

b. $\pi 3^2 h = 125$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the volume of a cylinder formula.

Accept alternative methods. Accept 4.43 $(4.42913\ldots)$ from using rounded answers in $h = \frac{125\times4.5}{127}$.

$h = 4.42\ (\mathrm{cm}) \quad (4.42097\ldots\ (\mathrm{cm}))$ ***(A1)(G2)***

c. $2\pi\times3\times4.5+\pi\times3\times4+\pi\times3^2$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for correct substitution into curved surface area of a cylinder formula, ***(M1)*** for correct substitution into the curved surface area of a cone formula, ***(M1)*** for adding the area of the base of the cylinder to the other two areas.

$= 151\ \mathrm{cm}^2 \quad (150.796\ldots\ \mathrm{cm}^2,\ 48\pi\ \mathrm{cm}^2)$ ***(A1)(G3)***

d. $\frac{150.796\ldots}{7}\times3$ ***(M1)(M1)***

Notes: Award ***(M1)*** for dividing their answer to (c) by 7, ***(M1)*** for multiplying by 3. Accept equivalent methods.

$= 64.63\ (\mathrm{ZAR})$ ***(A1)*****(ft)*****(G2)***

Notes: The ***(A1)*** is awarded for their correct answer, correctly rounded to 2 decimal places. Follow through from their answer to part (c). If rounded answer to part (c) is used the answer is 64.71 (ZAR).

e. $\frac{325}{13.03}$ ***(M1)***

Note: Award ***(M1)*** for dividing 325 by 13.03.

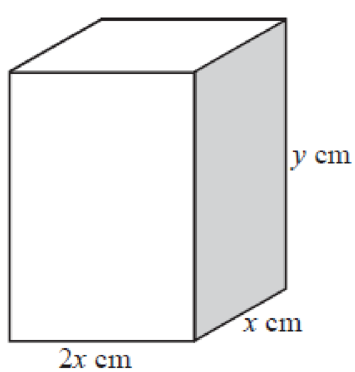
$= 24.94\ (\mathrm{EUR})$ ***(A1)(G2)***

Note: The ***(A1)*** is awarded for the correct answer rounded to 2 decimal places, unless already penalized in part (d).

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

A closed rectangular box has a height y cm and width x cm. Its length is twice its width. It has a fixed outer surface area of $300\ \mathrm{cm}^2$.



i.a. Factorise $3x^2 + 13x - 10$. [2]

i.b. Solve the equation $3x^2 + 13x - 10 = 0$. [2]

i.c. Consider a function $f(x) = 3x^2 + 13x - 10$. [2]

Find the equation of the axis of symmetry on the graph of this function.

i.d. Consider a function $f(x) = 3x^2 + 13x - 10$. [2]

Calculate the minimum value of this function.

ii.a. Show that $4x^2 + 6xy = 300$. [2]

ii.b. Find an expression for y in terms of x. [2]

ii.c. Hence show that the volume V of the box is given by $V = 100x - \frac{4}{3}x^3$. [2]

ii.d. Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

ii.e. (i) Hence find the value of x and of y required to make the volume of the box a maximum. [5]

(ii) Calculate the maximum volume.

Markscheme

i.a. $(3x - 2)(x + 5)$ ***(A1)(A1)***

[2 marks]

i.b. $(3x - 2)(x + 5) = 0$

$x = \frac{2}{3}$ or $x = -5$ ***(A1)*(ft)*(A1)*(ft)*(G2)***

[2 marks]

i.c. $x = \frac{-13}{6}$ (-2.17) ***(A1)(A1)*(ft)*(G2)***

Note: ***(A1)*** is for $x =$, ***(A1)*** for value. **(ft)** if value is half way between roots in (b).

[2 marks]

i.d. Minimum $y = 3\left(\frac{-13}{6}\right)^2 + 13\left(\frac{-13}{6}\right) - 10$ ***(M1)***

Note: ***(M1)*** for substituting their value of x from (c) into $f(x)$.

$= -24.1$ ***(A1)*(ft)*(G2)***

[2 marks]

ii.a. $\text{Area} = 2(2x)x + 2xy + 2(2x)y$ ***(M1)(A1)***

Note: ***(M1)*** for using the correct surface area formula (which can be implied if numbers in the correct place). ***(A1)*** for using correct numbers.

$300 = 4x^2 + 6xy$ ***(AG)***

Note: Final line must be seen or previous ***(A1)*** mark is lost.

[2 marks]

ii.b. $6xy = 300 - 4x^2$ ***(M1)***

$y = \frac{300-4x^2}{6x}$ *or* $\frac{150-2x^2}{3x}$ ***(A1)***

[2 marks]

ii.c. $\text{Volume} = x(2x)y$ ***(M1)***

$V = 2x^2\left(\frac{300-4x^2}{6x}\right)$ ***(A1)*(ft)**

$= 100x - \frac{4}{3}x^3$ **(AG)**

Note: Final line must be seen or previous ***(A1)*** mark is lost.

[2 marks]

ii.d. $\frac{\mathrm{d}V}{\mathrm{d}x} = 100 - \frac{12x^2}{3}$ or $100 - 4x^2$ ***(A1)(A1)***

Note: ***(A1)*** for each term.

[2 marks]

ii.e. *Unit penalty* ***(UP)*** *is applicable where indicated in the left hand column*

(i) For maximum $\frac{\mathrm{d}V}{\mathrm{d}x} = 0$ or $100 - 4x^2 = 0$ ***(M1)***

$x = 5$ ***(A1)*(ft)**

$y = \frac{300-4(5)^2}{6(5)}$ or $\left(\frac{150-2(5)^2}{3(5)}\right)$ ***(M1)***

$= \frac{20}{3}$ ***(A1)*(ft)**

(UP) (ii) $333\frac{1}{3}\ \text{cm}^3\ (333\ \text{cm}^3)$

Note: **(ft)** from their (e)(i) if working for volume is seen.

[5 marks]

Examiners report

i.a. Most candidates made a good attempt to factorise the expression.

i.b. Many gained both marks here from a correct answer or ft from the previous part.

i.c. Many used the formula correctly. Some forgot to put $x =$.

i.d. Most candidates found this value from their GDC.

ii.a. A good attempt was made to show the correct surface area.

ii.b. Many could rearrange the equation correctly.

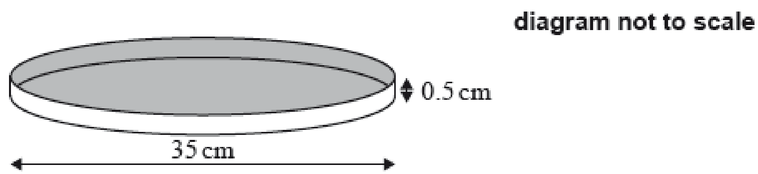
ii.c. Although this was not a difficult question it probably looked complicated for the candidates and it was often left out.

ii.d.Those who reached this length could usually manage the differentiation.

ii.e.(i) Many found the correct value of x but not of y.

(ii) This was well done and again the units were included in most scripts.

A pan, in which to cook a pizza, is in the shape of a cylinder. The pan has a diameter of 35 cm and a height of 0.5 cm.



A chef had enough pizza dough to exactly fill the pan. The dough was in the shape of a sphere.

The pizza was cooked in a hot oven. Once taken out of the oven, the pizza was placed in a dining room.

The temperature, P, of the pizza, in degrees Celsius, °C, can be modelled by

$$P(t) = a(2.06)^{-t} + 19,\ t \geqslant 0$$

where a is a constant and t is the time, in minutes, since the pizza was taken out of the oven.

When the pizza was taken out of the oven its temperature was 230 °C.

The pizza can be eaten once its temperature drops to 45 °C.

a. Calculate the volume of this pan. [3]

b. Find the radius of the sphere in cm, correct to one decimal place. [4]

c. Find the value of a. [2]

d. Find the temperature that the pizza will be 5 minutes after it is taken out of the oven. [2]

e. Calculate, to the nearest second, the time since the pizza was taken out of the oven until it can be eaten. [3]

f. In the context of this model, state what the value of 19 represents. [1]

Markscheme

a. $(V =)\ \pi \times (17.5)^2 \times 0.5$ ***(A1)(M1)***

Notes: Award ***(A1)*** for 17.5 (or equivalent) seen.

Award ***(M1)*** for correct substitutions into volume of a cylinder formula.

$= 481\ \text{cm}^3\ (481.056\ldots\ \text{cm}^3,\ 153.125\pi\ \text{cm}^3)$ ***(A1)(G2)***

[3 marks]

b. $\frac{4}{3} \times \pi \times r^3 = 481.056\ldots$ ***(M1)***

Note: Award ***(M1)*** for equating **their** answer to part (a) to the volume of sphere.

$r^3 = \frac{3 \times 481.056\ldots}{4\pi} \ (= 114.843\ldots)$ ***(M1)***

Note: Award ***(M1)*** for correctly rearranging so r^3 is the subject.

$r = 4.86074\ldots \text{ (cm)}$ ***(A1)*****(ft)*****(G2)***

Note: Award ***(A1)*** for correct unrounded answer seen. Follow through from part (a).

$= 4.9 \text{ (cm)}$ ***(A1)*****(ft)*****(G3)***

Note: The final ***(A1)*****(ft)** is awarded for rounding their unrounded answer to one decimal place.

[4 marks]

c. $230 = a(2.06)^0 + 19$ ***(M1)***

Note: Award ***(M1)*** for correct substitution.

$a = 211$ ***(A1)(G2)***

[2 marks]

d. $(P =)\ 211 \times (2.06)^{-5} + 19$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the function, $P(t)$. Follow through from part (c). The negative sign in the exponent is required for correct substitution.

$= 24.7$ (°C) $(24.6878\ldots$ (°C)) ***(A1)*****(ft)*****(G2)***

[2 marks]

e. $45 = 211 \times (2.06)^{-t} + 19$ ***(M1)***

Note: Award ***(M1)*** for equating 45 to the exponential equation and for correct substitution (follow through for their a in part (c)).

$(t =)\ 2.89711\ldots$ ***(A1)*****(ft)*****(G1)***

$174 \text{ (seconds) } (173.826\ldots \text{ (seconds)})$ ***(A1)*****(ft)*****(G2)***

Note: Award final ***(A1)*****(ft)** for converting their $2.89711\ldots$ minutes into seconds.

[3 marks]

f. the temperature of the (dining) room ***(A1)***

OR

the lowest final temperature to which the pizza will cool ***(A1)***

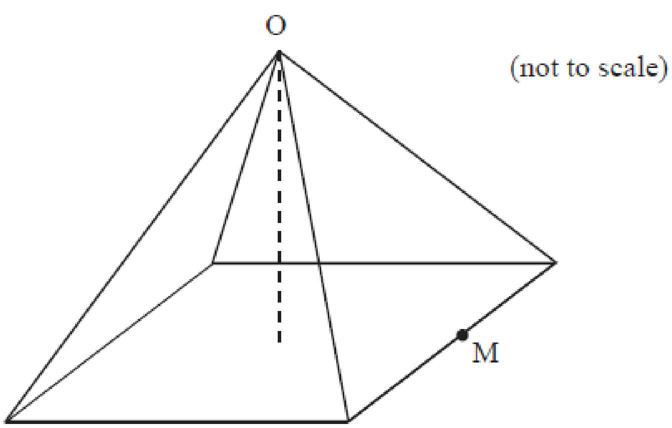
[1 mark]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

Amir needs to construct an isosceles triangle ABC whose area is $100\ \mathrm{cm}^2$. The equal sides, AB and BC, are $20\ \mathrm{cm}$ long.

Sylvia is making a square-based pyramid. Each triangle has a base of length $12\ \mathrm{cm}$ and a height of $10\ \mathrm{cm}$.



i.a. Angle ABC is acute. Show that the angle ABC is 30°. [2]

i.b. Find the length of AC. [3]

ii.a. Show that the **height** of the pyramid is $8\ \mathrm{cm}$. [2]

ii.b. M is the midpoint of the base of one of the triangles and O is the apex of the pyramid. [3]

Find the angle that the line MO makes with the base of the pyramid.

ii.c. Calculate the volume of the pyramid. [2]

ii.d. Daniel wants to make a rectangular prism with the same volume as that of Sylvia's pyramid. The base of his prism is to be a square of side $10\ \mathrm{cm}$. Calculate the height of the prism. [2]

Markscheme

i.a. $\frac{1}{2}20^2 \sin B = 100$ ***(M1)(A1)***

$B = 30^\circ$ ***(AG)***

Note: ***(M1)*** for correct substituted formula and ***(A1)*** for correct substitution. $B = 30^\circ$ must be seen or previous ***(A1)*** mark is lost.

[2 marks]

i.b. *Unit penalty* ***(UP)*** *is applicable where indicated in the left hand column.*

$\overline{\mathrm{AC}}^2 = 2 \times 20^2 - 2 \times 20^2 \times \cos 30^\circ$ ***(M1)(A1)***

(UP) $\overline{\mathrm{AC}} = 10.4 \text{ cm}$ ***(A1)(G2)***

Note: ***(M1)*** for using cosine rule, ***(A1)*** for correct substitution. Last ***(A1)*** is for the correct answer. Accept use of sine rule or any correct method e.g. $\mathrm{AC} = 2 \times 20 \sin 15^\circ$.

[3 marks]

ii.a. $x^2 + 6^2 = 10^2$ ***(A1)(M1)***

$x = 8 \text{ cm}$ ***(AG)***

Note: ***(A1)*** for 6 (or 36) seen and ***(M1)*** for using Pythagoras with correct substitution. $x = 8$ must be seen or previous ***(M1)*** mark is lost.

[2 marks]

ii.b. $\cos \beta = \frac{6}{10}$ ***(M1)(A1)***

$\beta = 53.1^\circ$ ***(A1)(G2)***

OR equivalent

Note: ***(M1)*** for use of trigonometric ratio with numbers from question. ***(A1)*** for use of correct numbers, and ***(A1)*** for correct answer.

[3 marks]

ii.c. *Unit penalty* ***(UP)*** *is applicable where indicated in the left hand column.*

$vol = \frac{12^2 \times 8}{3}$ ***(M1)***

(UP) $= 384 \text{ cm}^3$ ***(A1)(G2)***

Note: ***(M1)*** for correct formula and correct substitution, ***(A1)*** for correct answer.

[2 marks]

ii.d. *Unit penalty* ***(UP)*** *is applicable where indicated in the left hand column.*

Let h be the height

$10^2 h = 384$ ***(M1)***

(UP) $= 3.84 \text{ cm}$ ***(A1)*****(ft)*****(G2)***

Note: ***(M1)*** for correct formula and correct substitution, ***(A1)*** for correct answer. **(ft)** from answer to part (c).

[2 marks]

Examiners report

i.a. Many students did not write the units in their answers and were penalized with the UP in this question.

Part (a) was not very well answered. It looked as if the candidates did not understand the question. Many candidates did not draw a sketch of the triangle; this would have helped them to solve the question. Many candidates simply calculated the remaining angles of the triangle and showed that the sum was 180° . This was a clear example of the misunderstanding of the term "show that". Part (b) was well done though some candidates lost a mark for not giving the answer to the correct accuracy.

i.b. Many students did not write the units in their answers and were penalized with the UP in this question.

Part (a) was not very well answered. It looked as if the candidates did not understand the question. Many candidates did not draw a sketch of the triangle; this would have helped them to solve the question. Many candidates simply calculated the remaining angles of the triangle and showed that the sum was 180° . This was a clear example of the misunderstanding of the term "show that". Part (b) was well done though some candidates lost a mark for not giving the answer to the correct accuracy.

ii.a. Many students did not write the units in their answers and were penalized with the UP in this question.

The weaker candidates spent a lot of time in (a) using the wrong triangle to find half of the diagonal of the base. Finally they used Pythagoras theorem with the wrong numbers. Part (b) was well answered by most of the students. For the volume of the pyramid in (c) they used the correct formula though not always with the correct substitutions. To find the height of the prism in (d) the most common error was multiplying the volume of the prism by $\frac{1}{3}$. It seemed that many did not know the term 'prism'.

ii.b. Many students did not write the units in their answers and were penalized with the UP in this question.

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ii.d. Many students did not write the units in their answers and were penalized with the UP in this question.

The weaker candidates spent a lot of time in (a) using the wrong triangle to find half of the diagonal of the base. Finally they used Pythagoras theorem with the wrong numbers. Part (b) was well answered by most of the students. For the volume of the pyramid in (c) they used the correct formula though not always with the correct substitutions. To find the height of the prism in (d) the most common error was multiplying the volume of the prism by $\frac{1}{3}$. It seemed that many did not know the term 'prism'.

The quadrilateral ABCD shown below represents a sandbox. AB and BC have the same length. AD is 9 m long and CD is 4.2 m long. Angles ADC and ABC are 95° and 130° respectively.

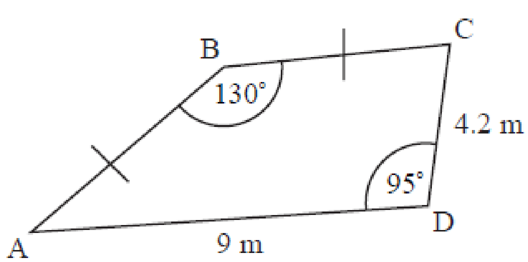

diagram not to scale

a. Find the length of AC. [3]

b. (i) Write down the size of angle BCA. [4]

(ii) Calculate the length of AB.

c. Show that the area of the sandbox is $31.1\ \text{m}^2$ correct to 3 s.f. [4]

d. The sandbox is a prism. Its edges are $40\ \text{cm}$ high. The sand occupies one third of the volume of the sandbox. Calculate the volume of sand in the sandbox. [3]

Markscheme

a. $\text{AC}^2 = 9^2 + 4.2^2 - 2 \times 9 \times 4.2 \times \cos 95^\circ$ ***(M1)(A1)***

$\text{AC} = 10.3\ \text{m}$ ***(A1)(G2)***

Note: ***(M1)*** for correct substituted formula and ***(A1)*** for correct substitution. If radians used answer is 6.59. Award at most ***(M1)(A1)(A0)***.

Note: The final ***A1*** is only awarded if the correct units are present; only penalize once for the lack of units or incorrect units.

b. (i) $\text{B}\hat{\text{C}}\text{A} = 25^\circ$ ***(A1)***

(ii) $\frac{\text{AB}}{\sin 25^\circ} = \frac{10.258\ldots}{\sin 130^\circ}$ ***(M1)(A1)***

$\text{AB} = 5.66\ \text{m}$ ***(A1)*****(ft)*****(G2)***

Note: ***(M1)*** for correct substituted formula and ***(A1)*** for correct substitution. ***(A1)*** for correct answer.

Follow through with angle $\text{B}\hat{\text{C}}\text{A}$ and their AC. Allow $\text{AB} = 5.68$ if $\text{AC} = 10.3$ used. If radians used answer is 0.938 (unreasonable answer). Award at most ***(M1)(A1)(A0)*****(ft)**.

OR

Using that ABC is isosceles

$\cos 25^\circ = \frac{\frac{1}{2} \times 10.258\ldots}{\text{AB}}$ (*or equivalent*) ***(A1)(M1)*****(ft)**

$\text{AB} = 5.66\ \text{m}$ ***(A1)*****(ft)*****(G2)***

Note: ***(A1)*** for $\frac{1}{2}$ of their AB seen, ***(M1)*** for correct trigonometric ratio and correct substitution, ***(A1)*** for correct answer. If $\frac{1}{2}\text{AB}$ seen and correct answer is given award ***(A1)(G1)***. Allow $\text{AB} = 5.68$ if $\text{AC} = 10.3$ used. If radians used answer is 3.32. Award ***(A1)(M1)(A1)*****(ft)**. If $\sin 65$ and radians used answer is 3.99. Award ***(A1)(M1)(A1)*****(ft)**.

Note: The final ***A1*** is only awarded in (ii) if the correct units are present; only penalize once for the lack of units or incorrect units.

c. Area $= \frac{1}{2} \times 9 \times 4.2 \times \sin 95^\circ + \frac{1}{2} \times (5.6592\ldots)^2 \times \sin 130^\circ$ ***(M1)(M1)*****(ft)*****(M1)***

$= 31.095\ldots = 31.1\ \text{m}^2$ (*correct to 3 s.f.*) ***(A1)(AG)***

Note: ***(M1)(M1)*** each for correct substitution in the formula of the area of each triangle, ***(M1)*** for adding both areas. ***(A1)*** for unrounded answer. Follow through with their length of AB but last mark is lost if they do not reach the correct answer.

d. Volume of sand $= \frac{1}{3}(31.09\ldots \times 0.4)$ ***(M1)(M1)***

$= 4.15\ \text{m}^3$ ***(A1)(G2)***

Note: ***(M1)*** for correct formula of volume of prism and for correct substitution, ***(M1)*** for multiplying by $\frac{1}{3}$ and last ***(A1)*** for correct answer only.

Note: The final ***A1*** is only awarded if the correct units are present; only penalize once for the lack of units or incorrect units.

Examiners report

a. It could have been written that the diagram was representing the plan of the sandbox. However, examiner’s comments did not find this lack of information an obstacle for the candidates.

   Overall the lengths of AC and AB were well done. Sine rule and cosine rule were in general well used. To find the length of AB many students used correctly right- angled trigonometry. The area of the sandbox was in general well done though some students did not gain the final mark due to premature rounding or for not showing the unrounded answer. The volume of the prism was poorly answered by the majority of the students. Most of the students did not use the correct formula. Very few candidates noticed that the value 40 was given in cm. It was good to see very few students losing marks for having their GDC setting in radians.

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Consider the function $f(x) = \frac{3}{4}x^4 - x^3 - 9x^2 + 20$.

a. Find $f(-2)$. [2]

b. Find $f'(x)$. [3]

c. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. [5]

   Using your answer to part (b), show that there is a second local minimum at $x = 3$.

d. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. [4]

   Sketch the graph of the function $f(x)$ for $-5 \leqslant x \leqslant 5$ and $-40 \leqslant y \leqslant 50$. Indicate on your sketch the coordinates of the y-intercept.

e. The graph of the function $f(x)$ has a local minimum at the point where $x = -2$. [2]

Write down the coordinates of the local maximum.

f. Let T be the tangent to the graph of the function $f(x)$ at the point $(2, -12)$. [2]

Find the gradient of T.

g. The line L passes through the point $(2, -12)$ and is perpendicular to T. [5]

L has equation $x + by + c = 0$, where b and $c \in \mathbb{Z}$.

Find

(i) the gradient of L;

(ii) the value of b and the value of c.

Markscheme

a. $\frac{3}{4}(-2)^4 - (-2)^3 - 9(-2)^2 + 20$ ***(M1)***

Note: Award ***(M1)*** for substituting $x = -2$ in the function.

$= 4$ ***(A1)(G2)***

Note: If the coordinates $(-2,\ 4)$ are given as the answer award, at most, ***(M1)(A0)***. If no working shown award ***(G1)***.

If $x = -2,\ y = 4$ seen then award full marks.

[2 marks]

b. $3x^3 - 3x^2 - 18x$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for each correct term, award at most ***(A1)(A1)(A0)*** if extra terms seen.

[3 marks]

c. $f'(3) = 3 \times (3)^3 - 3 \times (3)^2 - 18 \times 3$ ***(M1)***

Note: Award ***(M1)*** for substitution in their $f'(x)$ of $x = 3$.

$= 0$ ***(A1)***

OR

$3x^3 - 3x^2 - 18x = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their $f'(x)$ to zero.

$x = 3$ ***(A1)***

$f'(x_1) = 3 \times (x_1)^3 - 3 \times (x_1)^2 - 18 \times x_1 < 0$ where $0 < x_1 < 3$ ***(M1)***

Note: Award ***(M1)*** for substituting a value of x_1 in the range $0 < x_1 < 3$ into their f' and showing it is negative (decreasing).

$f'(x_2) = 3 \times (x_2)^3 - 3 \times (x_2)^2 - 18 \times x_2 > 0$ where $x_2 > 3$ ***(M1)***

Note: Award ***(M1)*** for substituting a value of x_2 in the range $x_2 > 3$ into their f' and showing it is positive (increasing).

OR

With or without a sketch:

Showing $f(x_1) > f(3)$ where $x_1 < 3$ and x_1 is close to 3. ***(M1)***

Showing $f(x_2) > f(3)$ where $x_2 > 3$ and x_2 is close to 3. ***(M1)***

Note: If a sketch of $f(x)$ is drawn **in this part of the question and** $x = 3$ is identified as a stationary point on the curve, then

(i) award, at most, ***(M1)(A1)(M1)(M0)*** if the stationary point has been found;

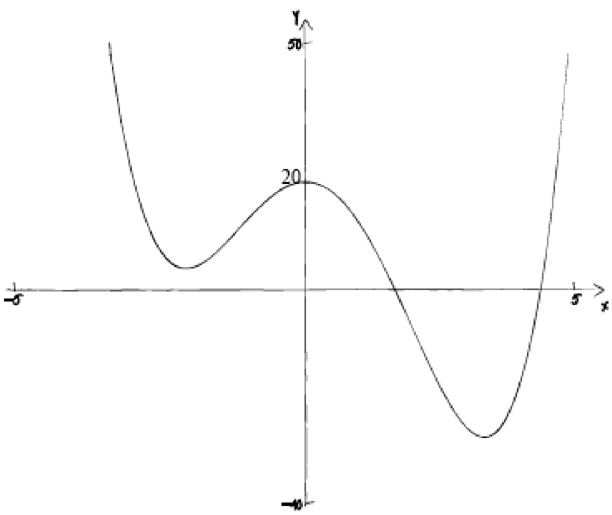
(ii) award, at most, ***(M0)(A0)(M1)(M0)*** if the stationary point has not been previously found.

Since the gradients go from negative (decreasing) through zero to positive (increasing) it is a local minimum ***(R1)(AG)***

Note: Only award ***(R1)*** if the first two marks have been awarded *ie* $f'(3)$ has been shown to be equal to 0.

[5 marks]

d.



(A1)(A1)(A1)(A1)

Notes: Award ***(A1)*** for labelled axes and indication of scale on both axes.

Award ***(A1)*** for smooth curve with correct shape.

Award ***(A1)*** for local minima in 2^{nd} and 4^{th} quadrants.

Award ***(A1)*** for *y* intercept $(0, 20)$ seen and labelled. Accept 20 on y-axis.

Do **not** award the third ***(A1)*** mark if there is a turning point on the x-axis.

If the derivative function is sketched then award, at most, ***(A1)(A0)(A0)(A0)***.

For a smooth curve (with correct shape) there should be **ONE** continuous thin line, no part of which is straight and no (one to many) mappings of x.

[4 marks]

e. $(0, 20)$ ***(G1)(G1)***

Note: If parentheses are omitted award ***(G0)(G1)***.

OR

$x = 0,\ y = 20$ ***(G1)(G1)***

Note: If the derivative function is sketched in part (d), award ***(G1)*****(ft)*****(G1)*****(ft)** for $(-1.12, 12.2)$.

[2 marks]

f. $f'(2) = 3(2)^3 - 3(2)^2 - 18(2)$ ***(M1)***

Notes: Award ***(M1)*** for substituting $x = 2$ into their $f'(x)$.

$= -24$ ***(A1)*****(ft)*****(G2)***

[2 marks]

g. (i) Gradient of perpendicular $= \frac{1}{24}$ $(0.0417, 0.041666\ldots)$ ***(A1)*****(ft)*****(G1)***

Note: Follow through from part (f).

(ii) $y + 12 = \frac{1}{24}(x - 2)$ ***(M1)(M1)***

Note: Award ***(M1)*** for correct substitution of $(2, -12)$, ***(M1)*** for correct substitution of their perpendicular gradient into equation of line.

OR

$-12 = \frac{1}{24} \times 2 + d$ ***(M1)***

$d = -\frac{145}{12}$

$y = \frac{1}{24}x - \frac{145}{12}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of $(2, -12)$ and gradient into equation of a straight line, ***(M1)*** for correct substitution of the perpendicular gradient and correct substitution of $d$into equation of line.

$b = -24,\ c = -290$ ***(A1)*****(ft)*****(A1)*****(ft)*****(G3)***

Note: Follow through from parts (f) and g(i).

To award **(ft)** marks, b and c must be integers.

Where candidate has used 0.042 from g(i), award ***(A1)*****(ft)** for -288.

[5 marks]

Examiners report

a. Surprisingly, a correct method for substituting the value of –2 into the given function led many candidates to a variety of incorrect answers. This suggests a poor handling of negative signs and/or poor use of the graphic display calculator. Many correct answers were seen in part (b) as candidates seemed to be well-drilled in the process of differentiation. Part (c), however, proved to be quite a discriminator. There were 5 marks for this part of the question and simply showing that $x - 3$ is a turning point was not sufficient for all of these marks. Many simply scored only two marks by substituting $x - 3$ into their answer to part (b). Once they had shown that there was a turning point at $x - 3$, candidates were not expected to use the second derivative but to show that the function decreases for $x < 3$ and increases for $x > 3$. Part (d) required a sketch which could have either been done on lined paper or on graph paper. The majority of candidates obtained at least two marks here with the most common errors seen being incomplete labelled axes and curves which were far from being smooth. In part (e), many candidates identified the correct coordinates for the two marks available. But for many candidates, this is where responses stopped as, in part (f), connecting the gradient function found in part (b) to the given coordinates proved problematic and only a significant minority of candidates were able to arrive at the required answer of –24. Indeed, there were many NR (no responses) to this part and the final part of the question. As many candidates found part (f) difficult, even more candidates found getting beyond the gradient of L very difficult indeed. A minority of candidates wrote down the gradient of their perpendicular but then did not seem to know where to proceed from there. Substituting their gradient for b and the coordinates (2, –12) into the equation $x + by + c = 0$ was a popular, but erroneous, method. It was a rare event indeed to see a script with a correct answer for this part of the question.

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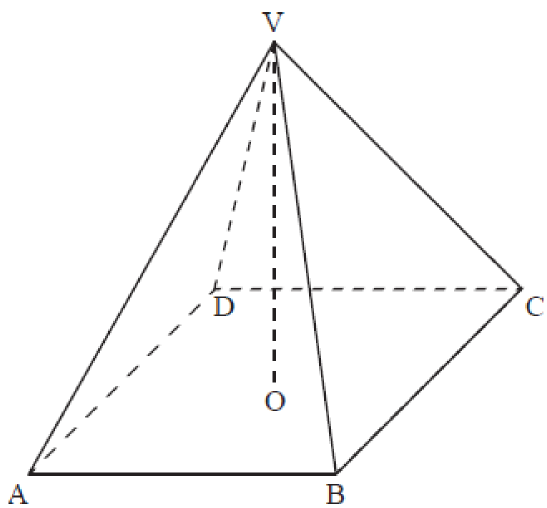
ABCDV is a solid glass pyramid. The base of the pyramid is a square of side 3.2 cm. The vertical height is 2.8 cm. The vertex V is directly above the centre O of the base.



a. Calculate the volume of the pyramid. [2]

b. The glass weighs 9.3 grams per cm^3. Calculate the weight of the pyramid. [2]

c. Show that the length of the sloping edge VC of the pyramid is 3.6 cm. [4]

d. Calculate the angle at the vertex, $\mathrm{B\hat{V}C}$. [3]

e. Calculate the total surface area of the pyramid. [4]

Markscheme

a. *Unit penalty* ***(UP)*** *is applicable in question parts (a), (b) and (e)* ***only.***

$\mathrm{V} = \frac{1}{3} \times 3.2^2 \times 2.8$ ***(M1)***

(M1) *for substituting in correct formula*

(UP) = 9.56 cm^3 ***(A1)(G2)***

[2 marks]

b. *Unit penalty **(UP)** is applicable in question parts (a), (b) and (e) **only**.*

9.56×9.3 ***(M1)***

(UP) = 88.9 grams ***(A1)*(ft)*(G2)***

[2 marks]

c. $\frac{1}{2}\text{base} = 1.6 \text{ seen}$ ***(M1)***

*award **(M1)** for halving base*

$\text{OC}^2 = 1.6^2 + 1.6^2 = 5.12$ ***(A1)***

*award **(A1)** for one correct use of Pythagoras*

$5.12 + 2.8^2 = 12.96 = \text{VC}^2$ ***(M1)***

*award **(M1)** for using Pythagoras again to find VC^2*

VC = 3.6 **AG**

*award **(A1)** for* 3.6 *obtained from* 12.96 *only (not* 12.95…*)* ***(A1)***

OR

$\text{AC}^2 = 3.2^2 + 3.2^2 = 20.48$ ***(A1)***

*award **(A1)** for one correct use of Pythagoras*

({\text{OC}} = \frac{1}{2} \sqrt{20.48}\) (= 2.26...) ***(M1)***

*award **(M1)** for halving AC*

$2.8^2 + (2.26...)^2 = \text{VC}^2 = 12.96$ ***(M1)***

*award **(M1)** for using Pythagoras again to find VC^2*

VC = 3.6 **AG** ***(A1)***

*award **(A1)** for* 3.6 *obtained from* 12.96 *only (not* 12.95…*)*

[4 marks]

d. $3.2^2 = 3.6^2 + 3.6^2 - 2 \times (3.6)(3.6)\cos \text{B}\hat{\text{V}}\text{C}$ ***(M1)(A1)***

$\text{B}\hat{\text{V}}\text{C} = 52.8^\circ$ *(no **(ft)** here)* ***(A1)(G2)***

*award **(M1)** for substituting in correct formula, **(A1)** for correct substitution*

OR

$\sin \text{B}\hat{\text{V}}\text{M} = \frac{1.6}{3.6}$ where *M* is the midpoint of BC ***(M1)(A1)***

$\text{B}\hat{\text{V}}\text{C} = 52.8^\circ$ *(no **(ft)** here)* ***(A1)***

[3 marks]

e. *Unit penalty **(UP)** is applicable in question parts (a), (b) and (e) **only**.*

$4 \times \frac{1}{2}(3.6)^2 \times \sin(52.8^\circ) + (3.2)^2$ ***(M1)(M1)(M1)***

*award **(M1)** for $\times 4$, **(M1)** for substitution in relevant triangle area, ($\frac{1}{2}(3.2)(2.8)$ gets **(M0)**)*

(M1) *for* $+(3.2)^2$

(UP) = 30.9 cm^2 (**(ft)** *from their (d))* ***(A1)*(ft)*(G2)***

[4 marks]

Examiners report

a. The volume of the pyramid and the weight were well done. Many candidates lost their unit penalty here. They had trouble showing that the sloping edge was 3.6 cm. The angle BVC was done well but not the total surface area. They knew that they needed four sides and the base, but finding the area of the triangle proved difficult for the less able candidates.

b. The volume of the pyramid and the weight were well done. Many candidates lost their unit penalty here. They had trouble showing that the sloping edge was 3.6 cm. The angle BVC was done well but not the total surface area. They knew that they needed four sides and the base, but finding the area of the triangle proved difficult for the less able candidates.

c. The volume of the pyramid and the weight were well done. Many candidates lost their unit penalty here. They had trouble showing that the sloping edge was 3.6 cm. The angle BVC was done well but not the total surface area. They knew that they needed four sides and the base, but finding the area of the triangle proved difficult for the less able candidates.

d. The volume of the pyramid and the weight were well done. Many candidates lost their unit penalty here. They had trouble showing that the sloping edge was 3.6 cm. The angle BVC was done well but not the total surface area. They knew that they needed four sides and the base, but finding the area of the triangle proved difficult for the less able candidates.

e. The volume of the pyramid and the weight were well done. Many candidates lost their unit penalty here. They had trouble showing that the sloping edge was 3.6 cm. The angle BVC was done well but not the total surface area. They knew that they needed four sides and the base, but finding the area of the triangle proved difficult for the less able candidates.

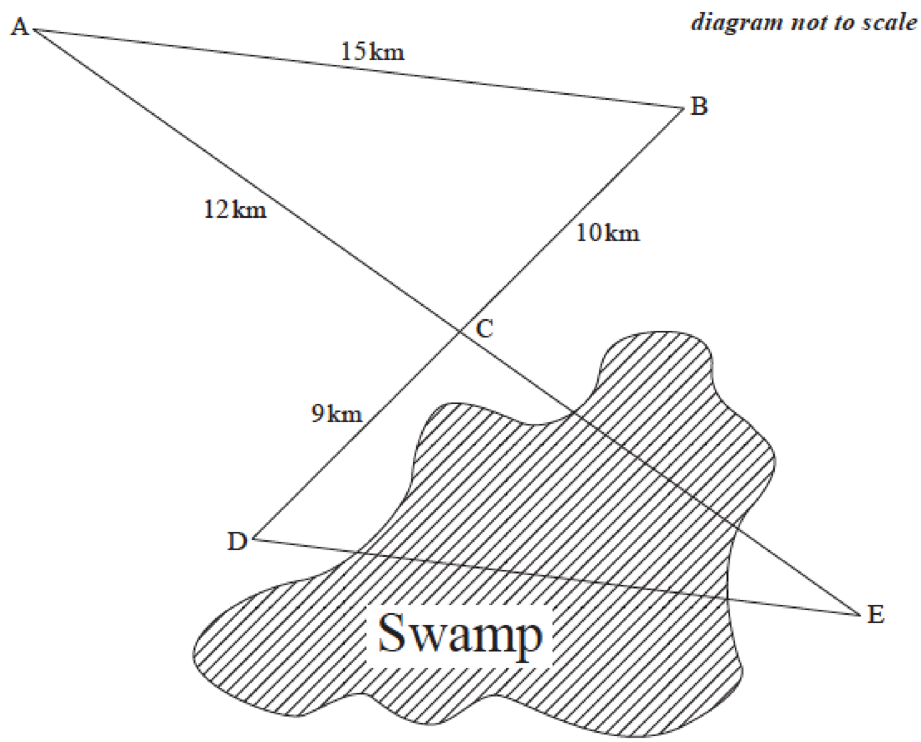
A surveyor has to calculate the area of a triangular piece of land, DCE.

The lengths of CE and DE cannot be directly measured because they go through a swamp.

AB, DE, BD and AE are straight paths. Paths AE and DB intersect at point C.

The length of AB is 15 km, BC is 10 km, AC is 12 km, and DC is 9 km.

The following diagram shows the surveyor’s information.



a. (i) Find the size of angle ACB. [4]

(ii) Show that the size of angle DCE is 85.5°, correct to one decimal place.

b. The surveyor measures the size of angle CDE to be twice that of angle DEC. [5]

(i) Using angle $\mathrm{DCE} = 85.5^\circ$, find the size of angle DEC.

(ii) Find the length of DE.

c. Calculate the area of triangle DEC. [4]

Markscheme

a. (i) $\cos \mathrm{A\hat{C}B} = \frac{10^2+12^2-15^2}{2\times10\times12}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted cosine rule,

(A1) for correct substitution.

$\mathrm{A\hat{C}B} = 85.5^\circ \quad (85.4593\ldots)$ ***(A1)(G2)***

(ii) $\mathrm{D\hat{C}E} = \mathrm{A\hat{C}B} \quad \text{and} \quad \mathrm{A\hat{C}B} = 85.5^\circ \quad (85.4593\ldots^\circ)$ ***(A1)***

OR

$\mathrm{B\hat{C}E} = 180^\circ - 85.5^\circ = 94.5^\circ \quad \text{and} \quad \mathrm{D\hat{C}E} = 180^\circ - 94.5^\circ = 85.5^\circ$ ***(A1)***

Notes: Both reasons must be seen for the ***(A1)*** to be awarded.

$\mathrm{D\hat{C}E} = 85.5^\circ$ ***(AG)***

b. (i) $\mathrm{D\hat{E}C} = \frac{180^\circ - 85.5^\circ}{3}$ ***(M1)***

$\hat{DEC} = 31.5°$ ***(A1)(G2)***

(ii) $\frac{\sin(31.5°)}{9} = \frac{\sin(85.5°)}{DE}$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substituted sine rule, ***(A1)*** for correct substitution.

$DE = 17.2\ (km)(17.1718\ldots).$ ***(A1)*****(ft)*****(G2)***

c. $0.5 \times 17.1718\ldots \times 9 \times \sin(63°)$ ***(A1)*****(ft)*****(M1)(A1)*****(ft)**

Note: Award ***(A1)*****(ft)** for 63 seen, ***(M1)*** for substituted triangle area formula, ***(A1)*****(ft)** for $0.5 \times 17.1718\ldots \times 9 \times \sin(\text{their angle CDE})$.

OR

$(\text{triangle height} =)\ 9 \times \sin(63°)$ ***(A1)*****(ft)*****(A1)*****(ft)**

$0.5 \times 17.1718\ldots \times 9 \times \sin(\text{their angle CDE})$ ***(M1)***

Note: Award ***(A1)*****(ft)** for 63 seen, ***(A1)*****(ft)** for correct triangle height with their angle CDE, ***(M1)*** for $0.5 \times 17.1718\ldots \times 9 \times \sin(\text{their angle CDE})$.

$= 68.9\ km^2 \quad (68.8509\ldots)$ ***(A1)*****(ft)*****(G3)***

Notes: Units are required for the last ***(A1)*****(ft)** mark to be awarded.

Follow through from parts (b)(i) and (b)(ii).

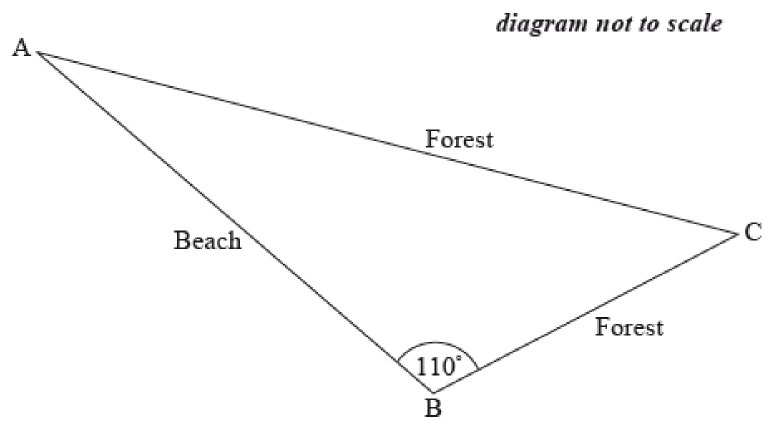
Follow through from their angle CDE **within this part of the question**.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

A cross-country running course consists of a beach section and a forest section. Competitors run from A to B, then from B to C and from C back to A.

The running course from A to B is along the beach, while the course from B, through C and back to A, is through the forest.
The course is shown on the following diagram.



Angle ABC is $110°$.

It takes Sarah 5 minutes and 20 seconds to run from A to B at a speed of $3.8\ \mathrm{ms}^{-1}$.

a. Using '*distance* = *speed* $\times$ *time*', show that the distance from A to B is 1220 metres correct to 3 significant figures. [2]

b. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [1]

Calculate the speed, in ms^{-1}, that Sarah runs from B to C.

c. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [3]

Calculate the distance, in metres, from **$\mathbf{C}$ to $\mathbf{A}$**.

d. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [2]

Calculate the total distance, in metres, of the cross-country running course.

e. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [3]

Find the size of angle BCA.

f. The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds. [3]

Calculate the area of the cross-country course bounded by the lines AB, BC and CA.

Markscheme

a. 3.8×320 ***(A1)***

Note: Award ***(A1)*** for 320 or equivalent seen.

$= 1216$ ***(A1)***

$= 1220\ (\mathrm{m})$ ***(AG)***

Note: Both unrounded and rounded answer must be seen for the final ***(A1)*** to be awarded.

[2 marks]

b. $\frac{850}{303}\ (\mathrm{ms}^{-1})\ (2.81, 2.80528\ldots)$ ***(A1)(G1)***

[1 mark]

c. $\mathrm{AC}^2 = 1220^2 + 850^2 - 2(1220)(850)\cos 110^\circ$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into cosine rule formula, ***(A1)*** for correct substitutions.

$\mathrm{AC} = 1710\ (\mathrm{m})\ (1708.87\ldots)$ ***(A1)(G2)***

Notes: Accept $1705\ (1705.33\ldots)$.

[3 marks]

d. $1220 + 850 + 1708.87\ldots$ ***(M1)***

$= 3780\ (\mathrm{m})\ (3778.87\ldots)$ ***(A1)*****(ft)*****(G1)***

Notes: Award ***(M1)*** for adding the three sides. Follow through from their answer to part (c). Accept $3771\ (3771.33\ldots)$.

[2 marks]

e. $\frac{\sin C}{1220} = \frac{\sin 110^\circ}{1708.87\ldots}$ ***(M1)(A1)*****(ft)**

Notes: Award ***(M1)*** for substitution into sine rule formula, ***(A1)*****(ft)** for correct substitutions. Follow through from their part (c).

$C = 42.1^\circ\ (42.1339\ldots)$ ***(A1)*****(ft)*****(G2)***

Notes: Accept $41.9^\circ, 42.0^\circ, 42.2^\circ, 42.3^\circ$.

OR

$\cos C = \frac{1708.87\ldots^2 + 850^2 - 1220^2}{2 \times 1708.87\ldots \times 850}$ ***(M1)(A1)*****(ft)**

Notes: Award ***(M1)*** for substitution into cosine rule formula, ***(A1)*****(ft)** for correct substitutions. Follow through from their part (c).

$C = 42.1^\circ\ (42.1339\ldots)$ ***(A1)*****(ft)*****(G2)***

Notes: Accept $41.2^\circ, 41.8^\circ, 42.4^\circ$.

[3 marks]

f. $\frac{1}{2} \times 1220 \times 850 \times \sin 110^\circ$ ***(M1)(A1)*****(ft)**

OR

$\frac{1}{2} \times 1708.87\ldots \times 850 \times \sin 42.1339\ldots^\circ$ ***(M1)(A1)*****(ft)**

OR

$\frac{1}{2} \times 1220 \times 1708.87\ldots \times \sin 27.8661\ldots^\circ$ ***(M1)(A1)*****(ft)**

Note: Award ***(M1)*** for substitution into area formula, ***(A1)*****(ft)** for correct substitution.

$= 487\,000\ \mathrm{m}^2\ (487\,230\ldots\ \mathrm{m}^2)$ ***(A1)*****(ft)*****(G2)***

Notes: The answer is $487\,000\ \mathrm{m}^2$, **units are required**.

Accept $486\,000\ \mathrm{m}^2\ (485\,633\ldots\ \mathrm{m}^2)$.

If workings are not shown and units omitted, award ***(G1)*** for $487\,000$ or $486\,000$.

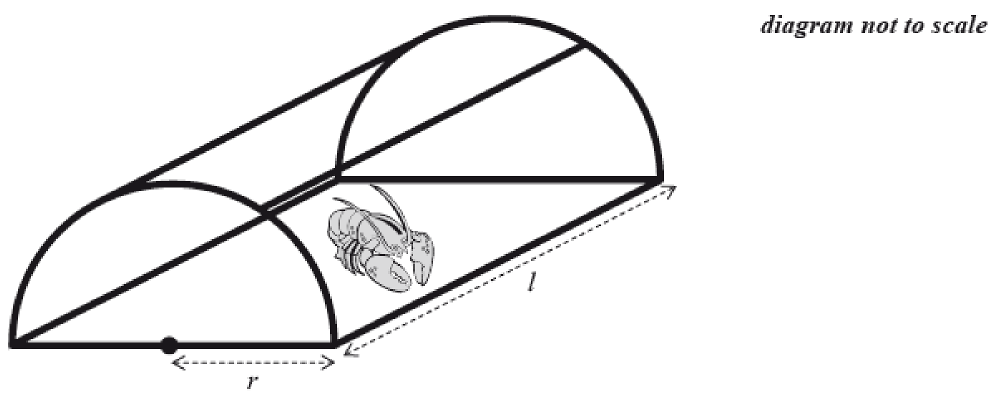
Follow through from parts (c) and (e).

[3 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

A lobster trap is made in the shape of half a cylinder. It is constructed from a steel frame with netting pulled tightly around it. The steel frame consists of a rectangular base, two semicircular ends and two further support rods, as shown in the following diagram.



The semicircular ends each have radius r and the support rods each have length l.
Let T be the total length of steel used in the frame of the lobster trap.

a. Write down an expression for T in terms of r, l and π. [3]

b. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. [3]

Write down an equation for the volume of the lobster trap in terms of r, l and π.

c. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. [2]

Show that $T = (2\pi + 4)r + \frac{6}{\pi r^2}$.

d. The volume of the lobster trap is $0.75\ \mathrm{m}^3$. [3]

Find $\frac{\mathrm{d}T}{\mathrm{d}r}$.

e. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Show that the value of r for which T is a minimum is 0.719 m, correct to three significant figures.

f. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Calculate the value of l for which T is a minimum.

g. The lobster trap is designed so that the length of steel used in its frame is a minimum. [2]

Calculate the minimum value of T.

Markscheme

a. $2\pi r + 4r + 4l$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for $2\pi r$ ("π" must be seen), ***(A1)*** for $4r$, ***(A1)*** for $4l$. Accept equivalent forms. Accept $T = 2\pi r + 4r + 4l$. Award a maximum of ***(A1)(A1)(A0)*** if extra terms are seen.

[3 marks]

b. $0.75 = \frac{\pi r^2 l}{2}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for their formula equated to 0.75, ***(A1)*** for l substituted into volume of cylinder formula, ***(A1)*** for volume of cylinder formula divided by 2.
If "π" not seen in part (a) accept use of 3.14 or greater accuracy. Award a maximum of ***(A1)(A1)(A0)*** if extra terms are seen.

[3 marks]

c. $T = 2\pi r + 4r + r\left(\frac{1.5}{\pi r^2}\right)$ ***(A1)*****(ft)*****(A1)***

$= (2\pi + 4)r + \frac{6}{\pi r^2}$ ***(AG)***

Notes: Award ***(A1)*****(ft)** for correct rearrangement of their volume formula in part (b) seen, award ***(A1)*** for the correct substituted formula for T. The final line must be seen, with no incorrect working, for this second ***(A1)*** to be awarded.

[2 marks]

d. $\frac{\mathrm{d}T}{\mathrm{d}r} = 2\pi + 4 - \frac{12}{\pi r^3}$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for $2\pi + 4$. ***(A1)*** for $\frac{-12}{\pi}$, ***(A1)*** for r^{-3}.
Accept 10.3 (10.2832…) for $2\pi + 4$, accept –3.82 –3.81971… for $\frac{-12}{\pi}$. Award a maximum of ***(A1)(A1)(A0)*** if extra terms are seen.

[3 marks]

e. $2\pi + 4 - \frac{12}{\pi r^3} = 0$ **OR** $\frac{\mathrm{d}T}{\mathrm{d}r} = 0$ ***(M1)***

Note: Award ***(M1)*** for setting their derivative equal to zero.

$r = 0.718843\ldots$ **OR** $\sqrt[3]{0.371452\ldots}$ **OR** $\sqrt[3]{\frac{12}{\pi(2\pi+4)}}$ **OR** $\sqrt[3]{\frac{3.81971}{10.2832\ldots}}$ ***(A1)***

$r = 0.719(\mathrm{m})$ ***(AG)***

Note: The rounded and unrounded or formulaic answers must be seen for the final ***(A1)*** to be awarded. The use of 3.14 gives an unrounded answer of $r = 0.719039\ldots$.

[2 marks]

f. $0.75 = \frac{\pi \times (0.719)^2 l}{2}$ ***(M1)***

Note: Award ***(M1)*** for substituting 0.719 into their volume formula. Follow through from part (b).

$l = 0.924(\mathrm{m})\ (0.923599\ldots)$ ***(A1)*****(ft)*****(G2)***

[2 marks]

g. $T = (2\pi + 4) \times 0.719 + \frac{6}{\pi(0.719)^2}$ ***(M1)***

Notes: Award ***(M1)*** for substituting 0.719 in their expression for T. Accept alternative methods, for example substitution of their l and 0.719 into their part (a) (for which the answer is 11.08961024). Follow through from their answer to part (a).

$= 11.1(\text{m})\ (11.0880\ldots)$ ***(A1)*****(ft)*****(G2)***

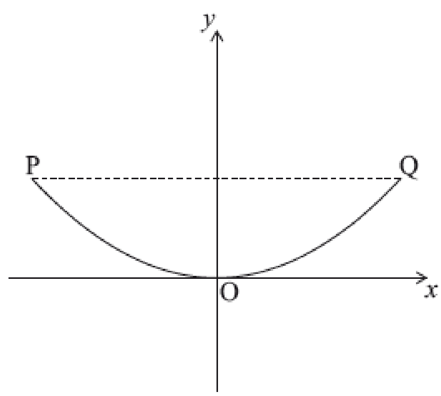
[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]

The front view of the edge of a water tank is drawn on a set of axes shown below.

The edge is modelled by $y = ax^2 + c$.



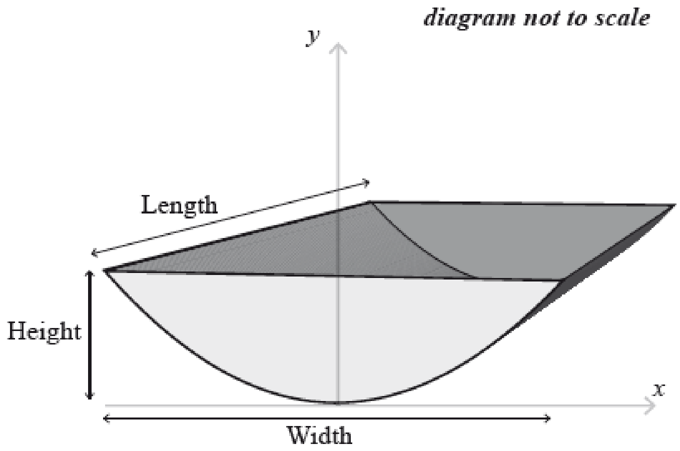
Point P has coordinates $(-3, 1.8)$, point O has coordinates $(0, 0)$ and point Q has coordinates $(3, 1.8)$.

a. Write down the value of c. [1]

b. Find the value of a. [2]

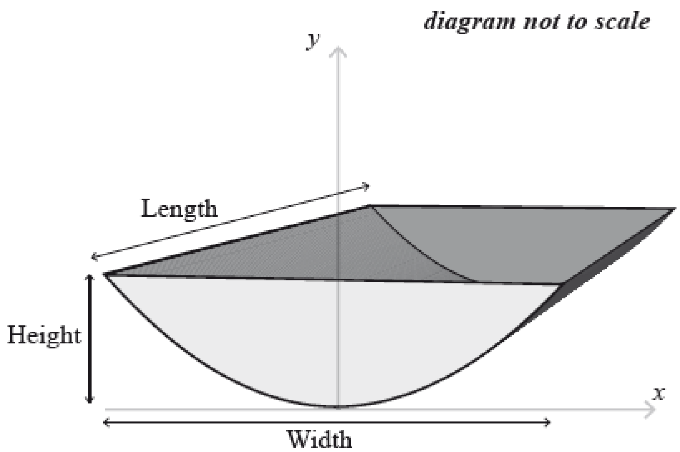
c. Hence write down the equation of the quadratic function which models the edge of the water tank. [1]

d. The water tank is shown below. It is partially filled with water. [2]



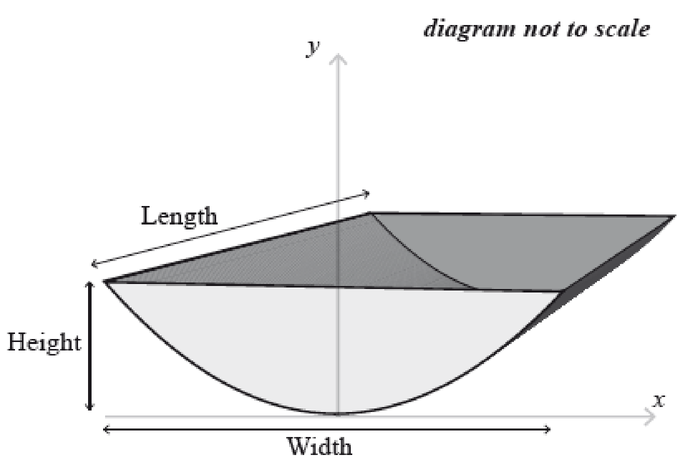
Calculate the value of y when $x = 2.4\text{ m}$.

e. The water tank is shown below. It is partially filled with water. [2]



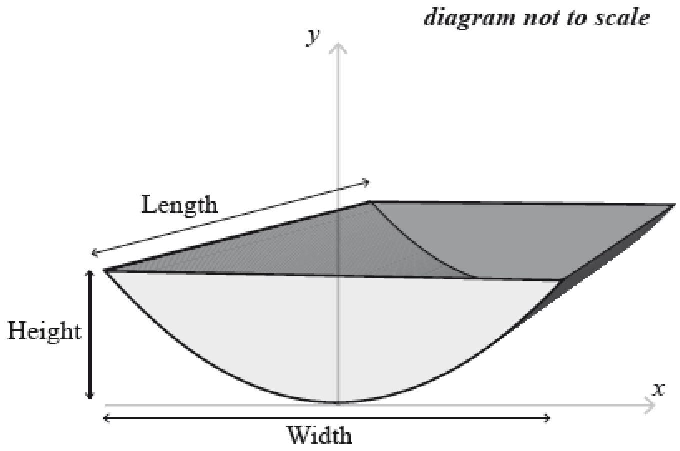
State what the value of x and the value of y represent for this water tank.

f. The water tank is shown below. It is partially filled with water. [2]



Find the value of x when the height of water in the tank is 0.9 m.

g. The water tank is shown below. It is partially filled with water. [2]



The water tank has a length of 5 m.

When the water tank is filled to a height of 0.9 m, the front cross-sectional area of the water is $2.55\ \mathrm{m}^2$.

(i) Calculate the volume of water in the tank.

The total volume of the tank is $36\ \mathrm{m}^3$.

(ii) Calculate the percentage of water in the tank.

Markscheme

a. 0 ***(A1)(G1)***

[1 mark]

b. $1.8 = a(3)^2 + 0$ ***(M1)***

OR

$1.8 = a(-3)^2 + 0$ ***(M1)***

Note: Award ***(M1)*** for substitution of $y = 1.8$ or $x = 3$ and their value of c into equation. 0 may be implied.

$a = 0.2\ \left(\frac{1}{5}\right)$ ***(A1)*****(ft)*****(G1)***

Note: Follow through from their answer to part (a).

Award ***(G1)*** for a correct answer only.

[2 marks]

c. $y = 0.2x^2$ ***(A1)*****(ft)**

Note: Follow through from their answers to parts (a) and (b).

Answer must be an equation.

[1 mark]

d. $0.2 \times (2.4)^2$ ***(M1)***

$= 1.15 \text{ (m)} \quad (1.152)$ ***(A1)*****(ft)*****(G1)***

Notes: Award ***(M1)*** for correctly substituted formula, ***(A1)*** for correct answer. Follow through from their answer to part (c).

Award ***(G1)*** for a correct answer only.

[2 marks]

e. y is the height ***(A1)***

positive value of x is half the width (*or equivalent*) ***(A1)***

[2 marks]

f. $0.9 = 0.2x^2$ ***(M1)***

Note: Award ***(M1)*** for setting their equation equal to 0.9.

$x = \pm 2.12 \text{ (m)} \quad \left(\pm\frac{3}{2}\sqrt{2},\ \pm\sqrt{4.5},\ \pm 2.12132\ldots\right)$ ***(A1)*****(ft)*****(G1)***

Note: Accept 2.12. Award ***(G1)*** for a correct answer only.

[2 marks]

g. (i) 2.55×5 ***(M1)***

Note: Award ***(M1)*** for correct substitution in formula.

$= 12.8 \text{ (m}^3) \quad \left(12.75 \text{ (m}^3)\right)$ ***(A1)(G2)***

[2 marks]

(ii) $\frac{12.75}{36} \times 100$ ***(M1)***

Note: Award ***(M1)*** for correct quotient multiplied by 100.

$= 35.4(\%) \ (35.4166\ldots)$ ***(A1)*****(ft)*****(G2)***

Note: Award ***(G2)*** for $35.6(\%)(35.5555\ldots(\%))$.

Follow through from their answer to part (g)(i).

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]

Consider the function $f(x) = \frac{96}{x^2} + kx$, where k is a constant and $x \neq 0$.

a. Write down $f'(x)$. [3]

b. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Show that $k = 3$.

c. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Find $f(2)$.

d. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Find $f'(2)$

e. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [3]

Find the equation of the normal to the graph of $y = f(x)$ at the point where $x = 2$.

Give your answer in the form $ax + by + d = 0$ where $a,\ b,\ d \in \mathbb{Z}$.

f. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [4]

Sketch the graph of $y = f(x)$, for $-5 \leqslant x \leqslant 10$ and $-10 \leqslant y \leqslant 100$.

g. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

Write down the coordinates of the point where the graph of $y = f(x)$ intersects the x-axis.

h. The graph of $y = f(x)$ has a local minimum point at $x = 4$. [2]

State the values of x for which $f(x)$ is decreasing.

Markscheme

a. $\frac{-192}{x^3} + k$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for -192, ***(A1)*** for x^{-3}, ***(A1)*** for k (only).

b. at local minimum $f'(x) = 0$ ***(M1)***

Note: Award ***(M1)*** for seeing $f'(x) = 0$ (may be implicit in their working).

$\frac{-192}{4^3} + k = 0$ ***(A1)***

$k = 3$ ***(AG)***

Note: Award ***(A1)*** for substituting $x = 4$ in their $f'(x) = 0$, provided it leads to $k = 3$. The conclusion $k = 3$ must be seen for the ***(A1)*** to be awarded.

c. $\frac{96}{2^2} + 3(2)$ ***(M1)***

Note: Award ***(M1)*** for substituting $x = 2$ and $k = 3$ in $f(x)$.

$= 30$ ***(A1)(G2)***

d. $\frac{-192}{2^3} + 3$ ***(M1)***

Note: Award ***(M1)*** for substituting $x = 2$ and $k = 3$ in their $f'(x)$.

$= -21$ ***(A1)*(ft)*(G2)***

Note: Follow through from part (a).

e. $y - 30 = \frac{1}{21}(x - 2)$ ***(A1)*(ft)*(M1)***

Notes: Award ***(A1)*(ft)** for their $\frac{1}{21}$ seen, ***(M1)*** for the correct substitution of their point and their normal gradient in equation of a line.

Follow through from part (c) and part (d).

OR

gradient of normal $= \frac{1}{21}$ ***(A1)*(ft)**

$30 = \frac{1}{21} \times 2 + c$ ***(M1)***

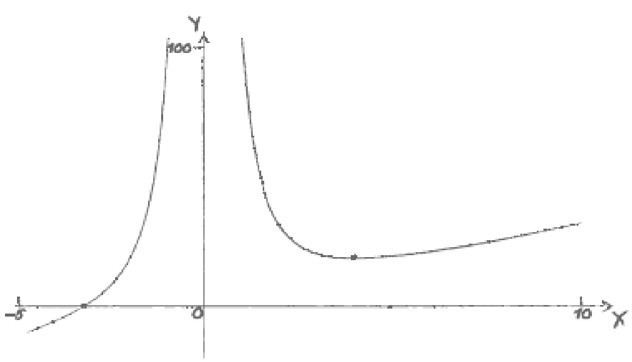
$c = 29\frac{19}{21}$

$y = \frac{1}{21}x + 29\frac{19}{21}$ $(y = 0.0476x + 29.904)$

$x - 21y + 628 = 0$ ***(A1)*(ft)*(G2)***

Notes: Accept equivalent answers.

f.



(A1)(A1)(A1)(A1)

Notes: Award ***(A1)*** for correct window (at least one value, other than zero, labelled on each axis), the axes must also be labelled; ***(A1)*** for a smooth curve with the correct shape (graph should not touch y-axis and should not curve away from the y-axis), on the given domain; ***(A1)*** for axis intercept in approximately the correct position (nearer -5 than zero); ***(A1)*** for local minimum in approximately the correct position (first quadrant, nearer the y-axis than $x = 10$).

If there is no scale, award a maximum of ***(A0)(A1)(A0)(A1)*** – the final ***(A1)*** being awarded for the zero and local minimum in approximately correct positions relative to each other.

g. $(-3.17,\ 0)$ $((-3.17480\ldots,\ 0))$ ***(G1)(G1)***

Notes: If parentheses are omitted award ***(G0)(G1)*(ft)**.

Accept $x = -3.17,\ y = 0$. Award ***(G1)*** for -3.17 seen.

h. $0 < x \leqslant 4$ or $0 < x < 4$ ***(A1)(A1)***

Notes: Award ***(A1)*** for correct end points of interval, ***(A1)*** for correct notation (note: lower inequality must be strict).

Award a maximum of ***(A1)(A0)*** if y or $f(x)$ used in place of x.

Examiners report

a. Differentiation of terms including negative indices remains a testing process; it will continue to be tested. There was, however, a noticeable improvement in responses compared to previous years. The parameter k was problematic for a number of candidates.

b. In part (b), the manipulation of the derivative to find the local minimum point caused difficulties for all but the most able; note that a GDC approach is not accepted in such questions and that candidates are expected to be able to apply the theory of the calculus as appropriate. Further, once a parameter is given, candidates are expected to use this value in subsequent parts.

c. Parts (c) and (d) were accessible and all but the weakest candidates scored well.

d. Parts (c) and (d) were accessible and all but the weakest candidates scored well.

e. Part (e) discriminated at the highest level; the gradient of the normal often was not used, the form of the answer not given correctly.

f. Curve sketching is a skill that most candidates find very difficult; axes must be labelled and some indication of the window must be present; care must be taken with the domain and the range; any asymptotic behaviour must be indicated. It was very rare to see sketches that attained full marks, yet this should be a skill that all can attain. There were many no attempts seen, yet some of these had correct answers to part (g).

g. Curve sketching is a skill that most candidates find very difficult; axes must be labelled and some indication of the window must be present; care must be taken with the domain and the range; any asymptotic behaviour must be indicated. It was very rare to see sketches that attained full marks, yet this should be a skill that all can attain. There were many no attempts seen, yet some of these had correct answers to part (g).

h. Part (h) was not well attempted in the main; decreasing (and increasing) functions is a testing concept for the majority.

The diagram below shows a square based right pyramid. ABCD is a square of side 10 cm. VX is the perpendicular height of 8 cm. M is the midpoint of BC.

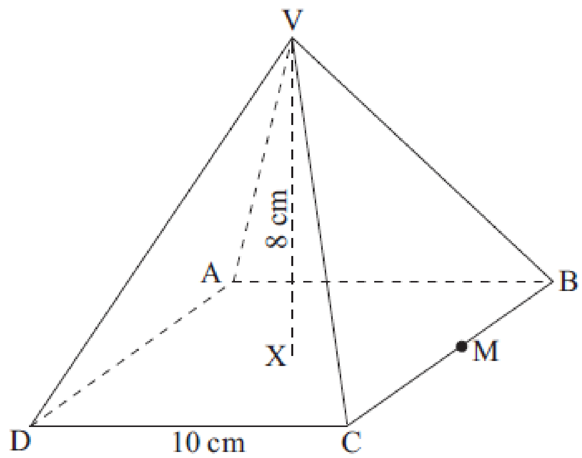


diagram not to scale

In a mountain region there appears to be a relationship between the number of trees growing in the region and the depth of snow in winter. A set of 10 areas was chosen, and in each area the number of trees was counted and the depth of snow measured. The results are given in the table below.

Number of trees (x)	**Depth of snow in cm (y)**
45	30
75	50
66	40
27	25
44	30
28	5
60	35
35	20
73	45
47	25

A path goes around a forest so that it forms the three sides of a triangle. The lengths of two sides are 550 m and 290 m. These two sides meet at an angle of 115°. A diagram is shown below.

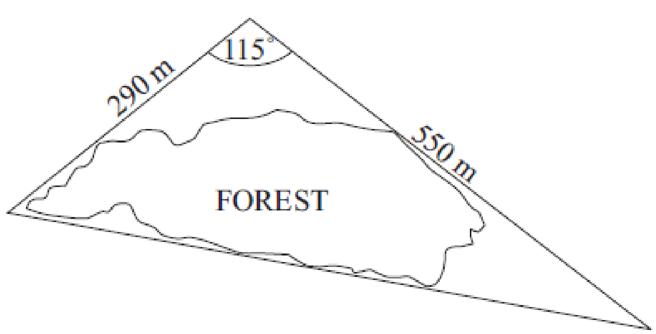


diagram not to scale

A, a Write down the length of XM. [1]

A, a Use your graphic display calculator to find the standard deviation of the number of trees. [1]

A, b Calculate the length of VM. [2]

A, c Calculate the angle between VM and ABCD. [2]

B, a Calculate the length of the third side of the triangle. Give your answer correct to the nearest 10 m. [4]

B, b Calculate the area enclosed by the path that goes around the forest. [3]

B, d Inside the forest a second path forms the three sides of another triangle named ABC. Angle BAC is 53°, AC is 180 m and BC is 230 m. [4]

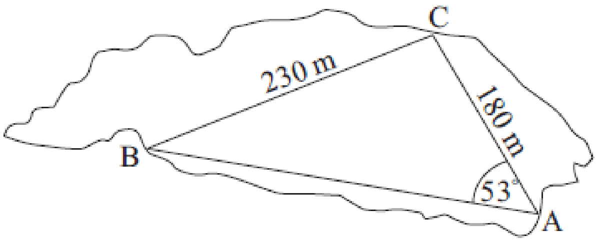


diagram not to scale

Calculate the size of angle ACB.

Markscheme

A, a ***UP applies in this question***

(UP) XM = 5 cm ***(A1)***

[1 mark]

A, a ii 16.8 ***(G1)***

[1 mark]

A, b ***UP applies in this question***

$VM^2 = 5^2 + 8^2$ ***(M1)***

Note: Award ***(M1)*** for correct use of Pythagoras Theorem.

(UP) VM = $\sqrt{89}$ = 9.43 cm ***(A1)*(ft)*(G2)***

[2 marks]

A, c $\tan \mathrm{VMX} = \frac{8}{5}$ ***(M1)***

Note: Other trigonometric ratios may be used.

$\mathrm{V\hat{M}X} = 58.0^\circ$ ***(A1)*(ft)*(G2)***

[2 marks]

B, a ***UP applies in this question***

$l^2 = 290^2 + 550^2 - 2 \times 290 \times 550 \times \cos 115°$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted cosine rule formula, ***(A1)*** for correct substitution.

$l = 722$ ***(A1)(G2)***

(UP) $= 720$ m ***(A1)***

Note: If 720 m seen without working award ***(G3)***.

The final ***(A1)*** is awarded for the correct rounding of their answer.

[4 marks]

B, b. ***UP applies in this question***

$\text{Area} = \frac{1}{2} \times 290 \times 550 \times \sin 115$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted correct formula ***(A1)*** for correct substitution.

(UP) $= 72\,300\ \text{m}^2$ ***(A1)(G2)***

[3 marks]

B, c. $\frac{180}{\sin \text{B}} = \frac{230}{\sin 53}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted sine rule formula, ***(A1)*** for correct substitution.

B = 38.7° ***(A1)(G2)***

$\text{A}\hat{\text{C}}\text{B} = 180 - (53^\circ + 38.7^\circ)$

$= 88.3^\circ$ ***(A1)*(ft)**

[4 marks]

Examiners report

A, a. This part proved accessible to the great majority of candidates. The common errors were (1) the inversion of the tangent ratio (2) the omission of the units and (3) the incorrect rounding of the answer; with 58° being all too commonly seen.

A, a. A straightforward question that saw many fine attempts. Given its nature – where much of the work was done on the GDC – it must be emphasised to candidates that incorrect entry of data into the calculator will result in considerable penalties; they must check their data entry most carefully.

The use of the inappropriate standard deviation was seen, but infrequently.

A, b. This part proved accessible to the great majority of candidates. The common errors were (1) the inversion of the tangent ratio (2) the omission of the units and (3) the incorrect rounding of the answer; with 58° being all too commonly seen.

A, c. This part proved accessible to the great majority of candidates. The common errors were (1) the inversion of the tangent ratio (2) the omission of the units and (3) the incorrect rounding of the answer; with 58° being all too commonly seen.

B, a. Again, this part proved accessible to the majority with a large number of candidates attaining full marks. However, there were also a number of candidates who seemed not to have been prepared in the use of trigonometry in non right-angled triangles. Also, failing to round the answer in (a) to the nearest 10m was a common omission.

B, b. Again, this part proved accessible to the majority with a large number of candidates attaining full marks. However, there were also a number of candidates who seemed not to have been prepared in the use of trigonometry in non right-angled triangles. Also, failing to round the answer in (a) to the nearest 10 m was a common omission.

B, c. Again, this part proved accessible to the majority with a large number of candidates attaining full marks. However, there were also a number of candidates who seemed not to have been prepared in the use of trigonometry in non right-angled triangles. Also, failing to round the answer in (a) to the nearest 10 m was a common omission.

Consider the function $f(x) = x^3 + \frac{48}{x}$, $x \neq 0$.

a. Calculate $f(2)$. [2]

b. Sketch the graph of the function $y = f(x)$ for $-5 \leqslant x \leqslant 5$ and $-200 \leqslant y \leqslant 200$. [4]

c. Find $f'(x)$. [3]

d. Find $f'(2)$. [2]

e. Write down the coordinates of the local maximum point on the graph of f . [2]

f. Find the range of f . [3]

g. Find the gradient of the tangent to the graph of f at $x = 1$. [2]

h. There is a second point on the graph of f at which the tangent is parallel to the tangent at $x = 1$. [2]

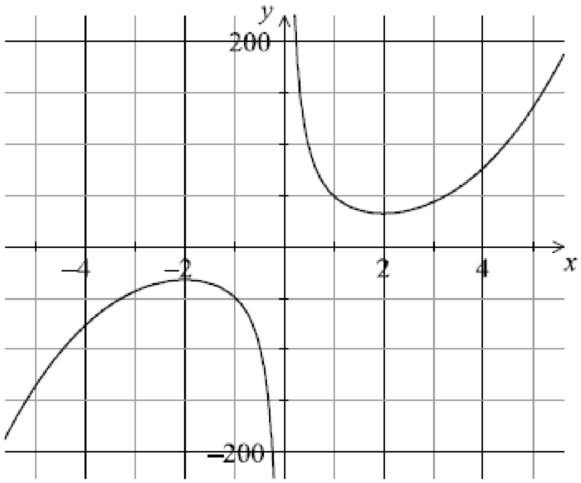
Find the x-coordinate of this point.

Markscheme

a. $f(2) = 2^3 + \frac{48}{2}$ ***(M1)***

$= 32$ ***(A1)(G2)***

[2 marks]

b. 

(A1) for labels and some indication of scale in an appropriate window
(A1) for correct shape of the two unconnected and smooth branches
(A1) for maximum and minimum in approximately correct positions
(A1) for asymptotic behaviour at y-axis ***(A4)***

Notes: Please be rigorous.
The axes need not be drawn with a ruler.
The branches must be smooth: a single continuous line that does not deviate from its proper direction.
The position of the maximum and minimum points must be symmetrical about the origin.
The y-axis must be an asymptote for both branches. Neither branch should touch the axis nor must the curve approach the asymptote then deviate away later.

[4 marks]

c. $f'(x) = 3x^2 - \frac{48}{x^2}$ ***(A1)(A1)(A1)***

Notes: Award ***(A1)*** for $3x^2$, ***(A1)*** for -48 , ***(A1)*** for x^{-2} . Award a maximum of ***(A1)(A1)(A0)*** if extra terms seen.

[3 marks]

d. $f'(2) = 3(2)^2 - \frac{48}{(2)^2}$ ***(M1)***

Note: Award ***(M1)*** for substitution of $x = 2$ into their derivative.

$= 0$ ***(A1)*(ft)*(G1)***

[2 marks]

e. $(-2, -32)$ or $x = -2, y = -32$ ***(G1)(G1)***

Notes: Award ***(G0)(G0)*** for $x = -32, y = -2$. Award at most ***(G0)(G1)*** if parentheses are omitted.

[2 marks]

f. $\{y \geqslant 32\} \cup \{y \leqslant -32\}$ ***(A1)(A1)*(ft)*(A1)*(ft)**

Notes: Award ***(A1)*(ft)** $y \geqslant 32$ or $y > 32$ seen, ***(A1)*(ft)** for $y \leqslant -32$ or $y < -32$, ***(A1)*** for weak (non-strict) inequalities used in both of the above.
Accept use of f in place of y. Accept alternative interval notation.
Follow through from their (a) and (e).
If domain is given award ***(A0)(A0)(A0)***.
Award ***(A0)(A1)*(ft)*(A1)*(ft)** for $[-200, -32]$, $[32, 200]$.
Award ***(A0)(A1)*(ft)*(A1)*(ft)** for $]-200, -32]$, $[32, 200[$.

[3 marks]

g. $f'(1) = -45$ ***(M1)(A1)*(ft)*(G2)***

Notes: Award ***(M1)*** for $f'(1)$ seen or substitution of $x = 1$ into their derivative. Follow through from their derivative if working is seen.

[2 marks]

h. $x = -1$ ***(M1)(A1)*(ft)*(G2)***

Notes: Award ***(M1)*** for equating their derivative to their -45 or for seeing parallel lines on their graph in the approximately correct position.

[2 marks]

Examiners report

a. As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

The majority of sketches were drawn sloppily and with little attention to detail. Teachers must impress on their students that a mathematical sketch is designed to illustrate the main points of a curve – the smooth nature by which it changes, any symmetries (reflectional or rotational), positions of turning points, intercepts with axes and the behaviour of a curve as it approaches an asymptote. There must also be some indication of the dimensions used for the “window”.

Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested.

It was also evident that some centres do not teach the differential calculus.

b. As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

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f. As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

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g. As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

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Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested.

It was also evident that some centres do not teach the differential calculus.

A shipping container is to be made with six rectangular faces, as shown in the diagram.

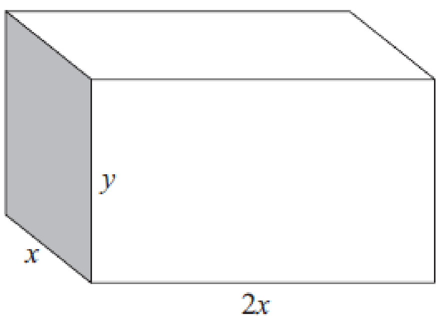

diagram not to scale

The dimensions of the container are

length $2x$
width x
height y.

All of the measurements are in metres. The total length of all twelve edges is 48 metres.

a. Show that $y = 12 - 3x$. [3]

b. Show that the volume V m^3 of the container is given by [2]

$V = 24x^2 - 6x^3$

c. Find $\frac{dV}{dx}$. [2]

d. Find the value of x for which V is a maximum. [3]

e. Find the maximum volume of the container. [2]

f. Find the length and height of the container for which the volume is a maximum. [3]

g. The shipping container is to be painted. One litre of paint covers an area of 15 m^2. Paint comes in tins containing four litres. [4]

Calculate the number of tins required to paint the shipping container.

Markscheme

a. $4(2x) + 4y + 4x = 48$ ***(M1)***

Note: Award ***(M1)*** for setting up the equation.

$12x + 4y = 48$ ***(M1)***

Note: Award ***(M1)*** for simplifying (can be implied).

$y = \frac{48-12x}{4}$ **OR** $3x + y = 12$ ***(A1)***

$y = 12 - 3x$ ***(AG)***

Note: The last line must be seen for the ***(A1)*** to be awarded.

[3 marks]

b. $V = 2x \times x \times (12 - 3x)$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into volume equation, ***(A1)*** for correct substitution.

$= 24x^2 - 6x^3$ ***(AG)***

Note: The last line must be seen for the ***(A1)*** to be awarded.

[2 marks]

c. $\frac{\mathrm{d}V}{\mathrm{d}x} = 48x - 18x^2$ ***(A1)(A1)***

Note: Award ***(A1)*** for each correct term.

[2 marks]

d. $48x - 18x^2 = 0$ ***(M1)(M1)***

Note: Award ***(M1)*** for using their derivative, ***(M1)*** for equating their answer to part (c) to 0.

OR

(M1) for sketch of $V = 24x^2 - 6x^3$, ***(M1)*** for the maximum point indicated ***(M1)(M1)***

OR

(M1) for sketch of $\frac{\mathrm{d}V}{\mathrm{d}x} = 48x - 18x^2$, ***(M1)*** for the positive root indicated ***(M1)(M1)***

$2.67 \left(\frac{24}{9},\ \frac{8}{3},\ 2.66666...\right)$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from their part (c).

[3 marks]

e. $V = 24 \times \left(\frac{8}{3}\right)^2 - 6 \times \left(\frac{8}{3}\right)^3$ ***(M1)***

Note: Award ***(M1)*** for substitution of their value from part (d) into volume equation.

$56.9(\mathrm{m}^3) \left(\frac{512}{9},\ 56.8888...\right)$ ***(A1)*****(ft)*****(G2)***

Note: Follow through from their answer to part (d).

[2 marks]

f. $\text{length} = \frac{16}{3}$ ***(A1)*****(ft)*****(G1)***

Note: Follow through from their answer to part (d). Accept 5.34 from use of 2.67

$\text{height} = 12 - 3 \times \left(\frac{8}{3}\right) = 4$ ***(M1)(A1)*****(ft)*****(G2)***

Notes: Award ***(M1)*** for substitution of their answer to part (d), ***(A1)(ft)*** for answer. Accept 3.99 from use of 2.67.

[3 marks]

g. $\text{SA} = 2 \times \frac{16}{3} \times 4 + 2 \times \frac{8}{3} \times 4 + 2 \times \frac{16}{3} \times \frac{8}{3}$ ***(M1)***

OR

$\text{SA} = 4\left(\frac{8}{3}\right)^2 + 6 \times \frac{8}{3} \times 4$ ***(M1)***

Note: Award ***(M1)*** for substitution of their values from parts (d) and (f) into formula for surface area.

92.4 (m^2) (92.4444...(m^2)) ***(A1)***

Note: Accept 92.5 (92.4622...) from use of 3 sf answers.

$\text{Number of tins} = \frac{92.4444...}{15 \times 4} (= 1.54)$ ***(M1)***

[4 marks]

Note: Award ***(M1)*** for division of their surface area by 60.

2 tins required ***(A1)*(ft)**

Note: Follow through from their answers to parts (d) and (f).

Examiners report

a. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(a) This was very poorly done. Most candidates had no idea what they were supposed to do here. Many tried to find values for x.

b. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(a) This was very poorly done. Most candidates had no idea what they were supposed to do here. Many tried to find values for x.

(b) Similar comment as for part (a) although more candidates made an attempt at finding the Volume.

c. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(c) This part was very well done.

d. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(d) Not many correct answers seen. Many candidates graphed the wrong equation and found 1.333 as their answer.

e. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(e) Some managed to gain follow through marks for this part.

f. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(f) Again here follow through marks were gained by those who attempted it.

g. Many candidates did not answer this question at all and others did not get past part (c). It was unclear if this was because they could not do the question or they ran out of time.

(g) Very few correct answers for the surface area were seen. Most candidates thought that there were 4 equal faces 2 xy and 2 faces xy. Some managed to get follow through marks for the last part if they divided by 60.

The diagram shows triangle ABC. Point C has coordinates (4, 7) and the equation of the line AB is $x + 2y = 8$.

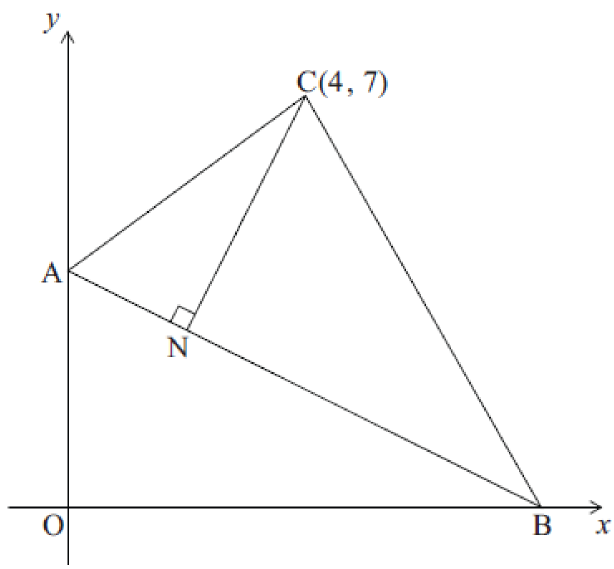

diagram not to scale

a.i. Find the coordinates of A. [1]

a.ii. Find the coordinates of B. [1]

b. Show that the distance between A and B is 8.94 correct to 3 significant figures. [2]

c.i. N lies on the line AB. The line CN is perpendicular to the line AB. [3]

Find the gradient of CN.

c.ii. N lies on the line AB. The line CN is perpendicular to the line AB. [2]

Find the equation of CN.

d. N lies on the line AB. The line CN is perpendicular to the line AB. [3]

Calculate the coordinates of N.

e. It is known that AC = 5 and BC = 8.06. [3]

Calculate the size of angle ACB.

f. It is known that AC = 5 and BC = 8.06. [3]

Calculate the area of triangle ACB.

Markscheme

a.i. A(0, 4) *Accept* $x = 0$, $y = 4$ ***(A1)***

[1 mark]

a.ii. B(8, 0) *Accept* $x = 8$, $y = 0$ ***(A1)*(ft)**

Note: Award ***(A0)*** if coordinates are reversed in (i) and ***(A1)*(ft)** in (ii).

[1 mark]

b. $\mathrm{AB} = \sqrt{8^2 + 4^2} = \sqrt{80}$ ***(M1)***

AB = 8.944 ***(A1)***

= 8.94 ***(AG)***

[2 marks]

c.i. $y = -0.5x + 4$ ***(M1)***

Gradient AB = –0.5 ***(A1)***

Note: Award ***(A2)*** if –0.5 seen.

OR

Gradient $\mathrm{AB} = \frac{(0-4)}{(8-0)}$ ***(M1)***

$= -\frac{1}{2}$ ***(A1)***

Note: Award ***(M1)*** for correct substitution in the gradient formula. Follow through from their answers to part (a).

Gradient CN = 2 ***(A1)*(ft)*(G2)***

Note: Special case: Follow through for gradient CN from their gradient AB.

[3 marks]

c.ii. CN: $y = 2x + c$

$7 = 2(4) + c$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in equation of a line.

$y = 2x - 1$ ***(A1)*(ft)*(G2)***

Note: Accept alternative forms for the equation of a line including $y - 7 = 2(x - 4)$. Follow through from their gradient in (i).

Note: If $c = -1$ seen but final answer is not given, award ***(A1)(d)***.

[2 marks]

d. $x + 2(2x - 1) = 8$ *or equivalent* ***(M1)***

N(2, 3) ($x = 2$, $y = 3$) ***(A1)(A1)*(ft)*(G3)***

Note: Award ***(M1)*** for attempt to solve simultaneous equations or a sketch of the two lines with an indication of the point of intersection.

[3 marks]

e. Cosine rule: $\cos(\mathrm{A\hat{C}B}) = \frac{5^2+8.06^2-8.944^2}{2\times5\times8.06}$ ***(M1)(A1)***

Note: Award ***(M1)*** for use of cosine rule with numbers from the problem substituted, ***(A1)*** for correct substitution.

$\mathrm{A\hat{C}B} = 82.9^\circ$ ***(A1)(G2)***

Note: If alternative right-angled trigonometry method used award ***(M1)*** for use of trig ratio in both triangles, ***(A1)*** for correct substitution of their values in each ratio, ***(A1)*** for answer.

Note: Accept 82.8° with use of 8.94.

[3 marks]

f. Area $\mathrm{ACB} = \frac{5\times8.06\sin(82.9)}{2}$ ***(M1)(A1)*(ft)**

Note: Award ***(M1)*** for substituted area formula, ***(A1)*** for correct substitution. Follow through from their angle in part (e).

OR

Area $\mathrm{ACB} = \frac{\mathrm{AB}\times\mathrm{CN}}{2} = \frac{8.94\times\sqrt{(4-2)^2+(7-3)^2}}{2}$ ***(M1)(M1)*(ft)**

Note: Award ***(M1)*** substituted area formula with their values, ***(M1)*** for substituted distance formula. Follow through from coordinates of N.

Area ACB = 20.0 ***(A1)*(ft)*(G2)***

Note: Accept 20

[3 marks]

Examiners report

a.i. This question had many correct solutions, but a large number of candidates were unable to follow the logical flow of the question to the end and many gave up. It should be pointed out to future candidates that parts (e) and (f) could be attempted independently from the rest and that care must be taken not to abandon hope too early in the longer questions of paper 2.

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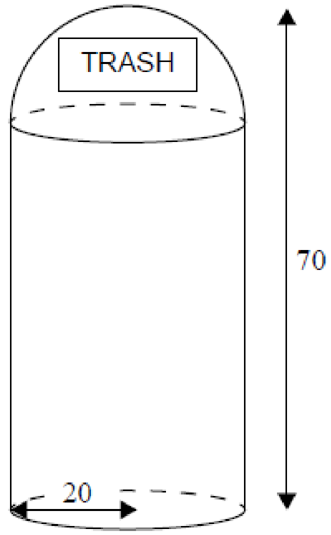
A manufacturer makes trash cans in the form of a cylinder with a hemispherical top. The trash can has a height of 70 cm. The base radius of both the cylinder and the hemispherical top is 20 cm.

diagram not to scale

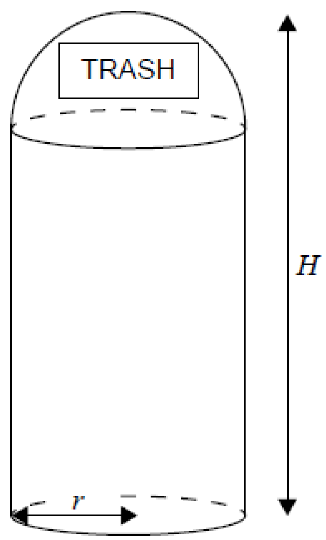


A designer is asked to produce a new trash can.

The new trash can will also be in the form of a cylinder with a hemispherical top.

This trash can will have a height of H cm and a base radius of r cm.

diagram not to scale



There is a design constraint such that $H + 2r = 110$ cm.

The designer has to maximize the volume of the trash can.

a. Write down the height of the cylinder. [1]

b. Find the total volume of the trash can. [4]

c. Find the height of the **cylinder**, h , of the new trash can, in terms of r. [2]

d. Show that the volume, V cm^3 , of the new trash can is given by [3]

$V = 110\pi r^3$.

e. Using your graphic display calculator, find the value of r which maximizes the value of V. [2]

f. The designer claims that the new trash can has a capacity that is at least 40% greater than the capacity of the original trash can. [4]

State whether the designer’s claim is correct. Justify your answer.

Markscheme

a. 50 (cm) ***(A1)***

[1 mark]

b. $\pi \times 50 \times 20^2 + \frac{1}{2} \times \frac{4}{3} \times \pi \times 20^3$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for their correctly substituted volume of cylinder, ***(M1)*** for correctly substituted volume of sphere formula, ***(M1)*** for halving the substituted volume of sphere formula. Award at most ***(M1)(M1)(M0)*** if there is no addition of the volumes.

$= 79600\ \left(\text{cm}^3\right)\ \left(79587.0\ldots\left(\text{cm}^3\right),\ \frac{76000}{3}\pi\right)$ ***(A1)*****(ft)** ***(G3)***

Note: Follow through from part (a).

[4 marks]

c. $h = H - r$ (or equivalent) ***OR*** $H = 110 - 2r$ ***(M1)***

Note: Award ***(M1)*** for writing h in terms of H and r or for writing H in terms of r.

(h =) $110 - 3r$ ***(A1) (G2)***

[2 marks]

d. $(V =)\ \frac{2}{3}\pi r^3 + \pi r^2 \times (110 - 3r)$ ***(M1)(M1)(M1)***

Note: Award ***(M1)*** for volume of hemisphere, ***(M1)*** for correct substitution of their h into the volume of a cylinder, ***(M1)*** for addition of two correctly substituted volumes leading to the given answer. Award at most ***(M1)(M1)(M0)*** for subsequent working that does not lead to the given answer. Award at most ***(M1)(M1)(M0)*** for substituting $H = 110 - 2r$ as their h.

$V = 110\pi r^2 - \frac{7}{3}\pi r^3$ ***(AG)***

[3 marks]

e. (r =) 31.4 (cm) (31.4285… (cm)) ***(G2)***

OR

$(\pi)\left(220r - 7r^2\right) = 0$ ***(M1)***

Note: Award ***(M1)*** for setting the correct derivative equal to zero.

(r =) 31.4 (cm) (31.4285… (cm)) ***(A1)***

[2 marks]

f. $(V =)\ 110\pi(31.4285\ldots)^3 - \frac{7}{3}\pi(31.4285\ldots)^3$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of their 31.4285… into the given equation.

= 114000 (113781…) ***(A1)*****(ft)**

Note: Follow through from part (e).

(increase in capacity =) $\frac{113.781\ldots - 79587.0\ldots}{79587.0\ldots} \times 100 = 43.0\ (\%)$ ***(R1)*****(ft)**

Note: Award ***(R1)*****(ft)** for finding the correct percentage increase from their two volumes.

OR

1.4 × 79587.0… = 111421.81… ***(R1)*****(ft)**

Note: Award ***(R1)*****(ft)** for finding the capacity of a trash can 40% larger than the original.

Claim is correct ***(A1)*****(ft)**

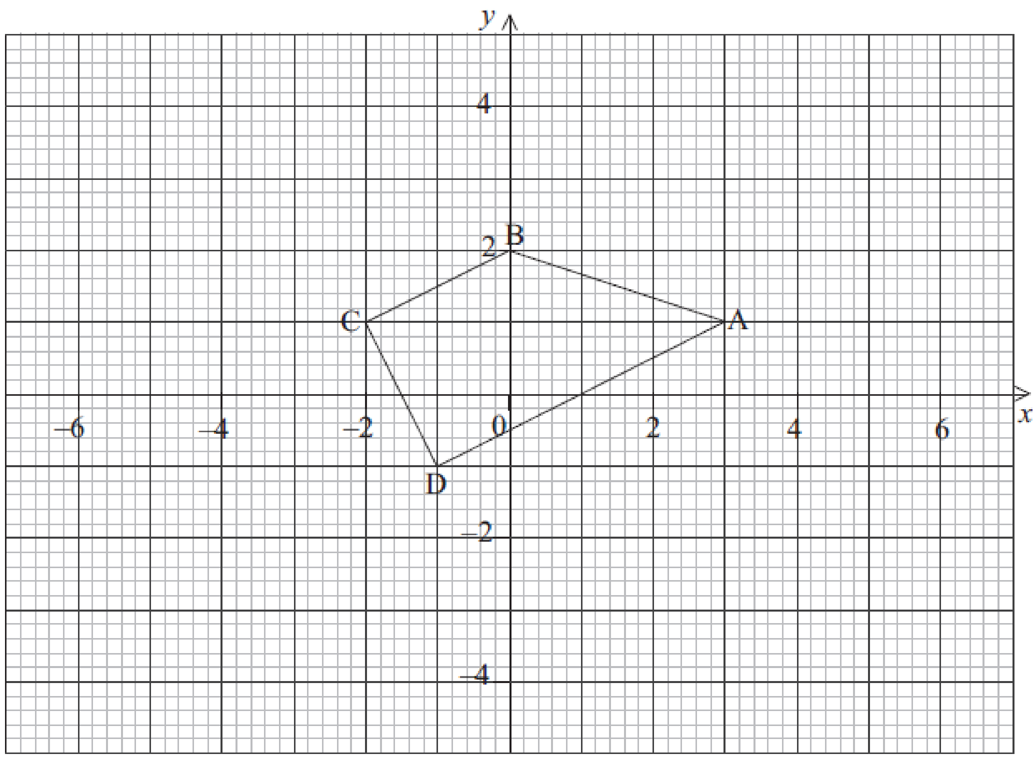
Note: Follow through from parts (b), (e) and within part (f). The final ***(R1)(A1)*****(ft)** can be awarded for their correct reason and conclusion. Do not award ***(R0)(A1)*****(ft)**.

[4 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

The vertices of quadrilateral ABCD as shown in the diagram are A (3, 1), B (0, 2), C (–2, 1) and D (–1, –1).



a. Calculate the gradient of line CD. [2]

b. Show that line AD is perpendicular to line CD. [2]

c. Find the equation of line CD. Give your answer in the form $ax + by = c$ where $a,\ b,\ c \in \mathbb{Z}$. [3]

d. Lines AB and CD intersect at point E. The equation of line AB is $x + 3y = 6$. [2]

Find the coordinates of E.

e. Lines AB and CD intersect at point E. The equation of line AB is $x + 3y = 6$. [2]

Find the distance between A and D.

f. The distance between D and E is $\sqrt{20}$. [2]

Find the area of triangle ADE.

Markscheme

a. Gradient of CD $= \frac{1-(-1)}{-2-(-1)}$ ***(M1)***

$= -2$ ***(A1)(G2)***

Note: Award ***(M1)*** for correct substitution in gradient formula.

[2 marks]

b. Gradient of AD $= \frac{1}{2}$ ***(A1)***

$-2 \times \frac{1}{2} = -1$ or $\frac{1}{2}$ is negative reciprocal of –2 ***(M1)***

Hence AD is perpendicular for CD. ***(AG)***

Note: Last line must be seen for the ***(M1)*** to be awarded.

[2 marks]

c. $y = -2x - 3$ ***(A1)*****(ft)*****(A1)*****(ft)**

Note: Award ***(A1)*****(ft)** for their (a), ***(A1)*****(ft)** for –3.

If part (a) incorrect award ***(A1)*****(ft)** for their *y*-intercept only if working is seen.

OR

$y - 1 = -2(x + 2)$ ***(A1)*****(ft)*****(A1)***

OR

$y + 1 = -2(x + 1)$ ***(A1)*****(ft)*****(A1)***

Note: Award ***(A1)*****(ft)** for their (a), ***(A1)*** for correct substitution of point.

$2x + y = -3$ ***(A1)*****(ft)**

Note: The final ***(A1)*****(ft)** is for their equation in the stated form.

[3 marks]

d. E (–3, 3) (Accept x = –3, y = 3) ***(G2)***

OR

Award ***(M1)*** for solving the pair of simultaneous equations by hand. ***(A1)*****(ft)** for correct answer, **(ft)** from their (c). ***(M1)(A1)*****(ft)**

OR

Award ***(M1)*** for having extended the lines in their own graph seen drawn on answer paper. ***(A1)*** for correct answer. ***(M1)(A1)***

Note: Missing coordinate brackets receive ***(G1)(G0)*** or ***(M1)(A0)***.

[2 marks]

e. Distance between A and D = $\sqrt{4^2+2^2}$ ***(M1)***

$= \sqrt{20}$ **OR** $2\sqrt{5}$ **OR** 4.47 (3 *s.f.*) ***(A1)(G2)***

Note: Award ***(M1)*** for correct substitution into the distance formula, ***(A1)*** for correct answer.

[2 marks]

f. Area of ADE = $\frac{1}{2}\sqrt{20}\times\sqrt{20}$ ***(M1)***

= 10 ***(A1)*(ft)*(G2)***

Follow through from (e).

[2 marks]

Examiners report

a. This was well done overall. Almost all students could calculate the gradient of the straight line. Gradient of perpendicular line was found, but some candidates failed to communicate the requirement, in terms of gradients for two lines to be perpendicular (Example: They are perpendicular because their gradients are opposite and reciprocal or they are perpendicular because the product of their gradients is -1)

Distance between points and area of triangle was answered well by most candidates. Both formulae for the area of the triangle were correctly used.

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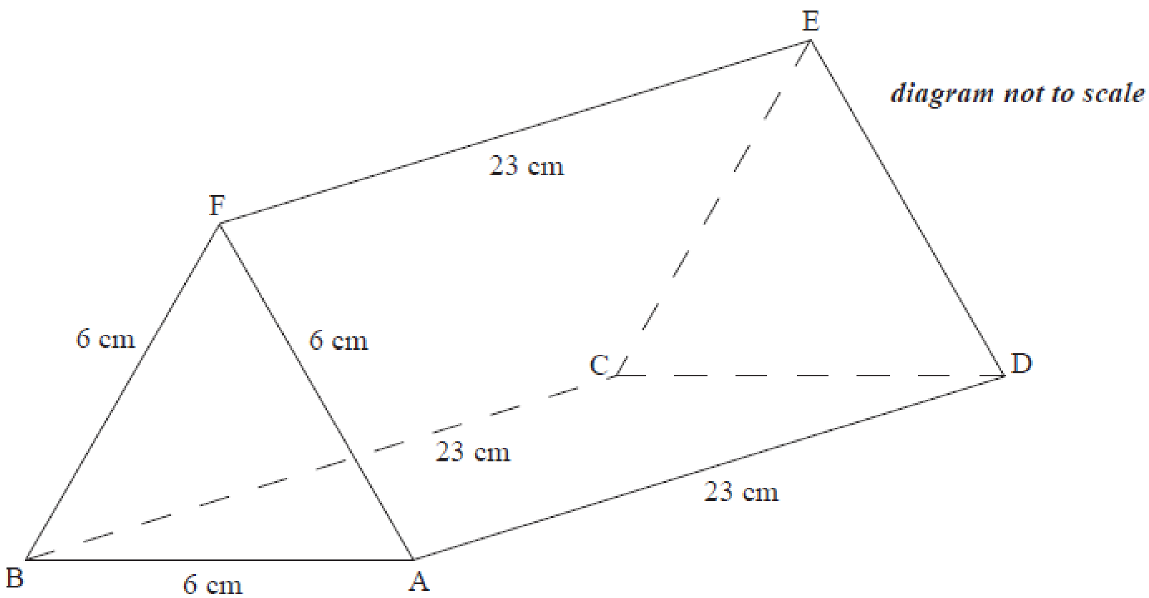
A chocolate bar has the shape of a triangular right prism ABCDEF as shown in the diagram. The ends are equilateral triangles of side 6 cm and the length of the chocolate bar is 23 cm.



a, i. Write down the size of angle BAF. [1]

a, ii. Hence or otherwise find the area of the triangular end of the chocolate bar. [3]

b. Find the total surface area of the chocolate bar. [3]

c. It is known that 1 cm^3 of this chocolate weighs 1.5 g. Calculate the weight of the chocolate bar. [3]

d. A different chocolate bar made with the same mixture also has the shape of a triangular prism. The ends are triangles with sides of length 4 cm, 6 cm and 7 cm. [3]

Show that the size of the angle between the sides of 6 cm and 4 cm is 86.4° correct to 3 significant figures.

e. The weight of this chocolate bar is 500 g. Find its length. [4]

Markscheme

a, i. $60°$ ***(A1)***

[1 mark]

a, ii. ***Unit penalty (UP) applies in this part***

$\text{Area} = \frac{6\times6\times\sin 60°}{2}$ ***(M1)(A1)***

(UP) = 15.6 cm^2 $(9\sqrt{3})$ ***(A1)(ft)(G2)***

Note: Award ***(M1)*** for substitution into correct formula, ***(A1)*** for correct values. Accept alternative correct methods.

[3 marks]

b. ***Unit penalty (UP) applies in this part***

$\text{Surface Area} = 15.58 \times 2 + 23 \times 6 \times 3$ ***(M1)(M1)***

Note: Award ***(M1)*** for two terms with 2 and 3 respectively, ***(M1)*** for 23×6 (138).

(UP) Surface Area = 445 cm^2 ***(A1)(ft)(G2)***

[3 marks]

c. ***Unit penalty (UP) applies in this part***

$\text{weight} = 1.5 \times 15.59 \times 23$ ***(M1)(M1)***

Note: Award ***(M1)*** for finding the volume, ***(M1)*** for multiplying their volume by 1.5.

(UP) weight = 538 g ***(A1)(ft)(G3)***

[3 marks]

d. $\cos\alpha = \frac{4^2+6^2-7^2}{2\times4\times6}$ ***(M1)(A1)***

Note: Award ***(M1)*** for using cosine rule with values from the problem, ***(A1)*** for correct substitution.

$\alpha = 86.41\ldots$ ***(A1)***

$\alpha = 86.4°$ ***(AG)***

Note: 86.41… must be seen for final ***(A1)*** to be awarded.

[3 marks]

e. ***Unit penalty (UP) applies in this part***

$l \times \frac{4\times6\times\sin 86.4^\circ}{2} \times 1.5 = 500$ ***(M1)(A1)(M1)***

Notes: Award ***(M1)*** for finding an expression for the volume, ***(A1)*** for correct substitution, ***(M1)*** for multiplying the volume by 1.5 and equating to 500, or for equating the volume to $\frac{500}{1.5}$.

If formula for volume is not correct but consistent with that in (c) award at most ***(M1)(A0)*****(ft)*****(M1)(A0)***.

(UP) *l* = 27.8 cm ***(A1)(G3)***

[4 marks]

Examiners report

a, i. It was pleasing to show candidate working throughout this question. Follow through marks could be awarded when incorrect answers were given. Many candidates incorrectly calculated the weight of the chocolate bar by multiplying the surface area by $1.5g$. Also a large number of students incorrectly used the formula for the volume of a pyramid rather than for a prism. Most candidates were successful in their use of the cosine rule but did not give the answer before it was rounded to 86.4, resulting in the loss of the final *A* mark. The last part acted as a clear discriminator, very few students were able to find the correct length of the new chocolate bar. Most students used units correctly.

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Nadia designs a wastepaper bin made in the shape of an **open** cylinder with a volume of 8000 cm^3.

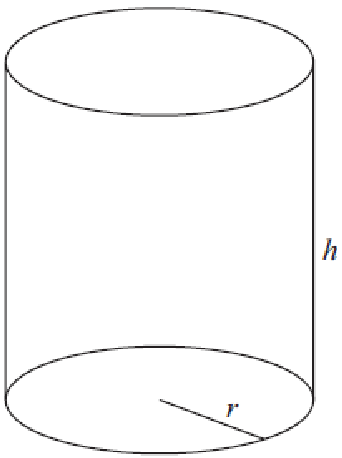


diagram not to scale

Nadia decides to make the radius, r , of the bin 5 cm.

Merryn also designs a cylindrical wastepaper bin with a volume of 8000 cm^3. She decides to fix the radius of its base so that the **total external surface area** of the bin is minimized.

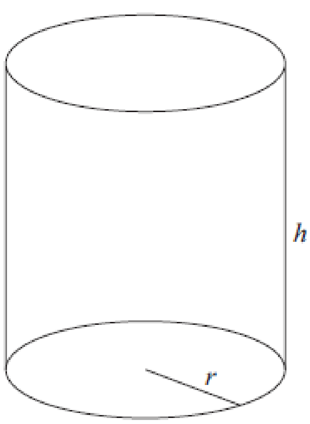


diagram not to scale

Let the radius of the base of Merryn’s wastepaper bin be r , and let its height be h .

a. Calculate [7]

(i) the area of the base of the wastepaper bin;

(ii) the height, h , of Nadia's wastepaper bin;

(iii) the total **external** surface area of the wastepaper bin.

b. State whether Nadia's design is practical. Give a reason. [2]

c. Write down an equation in h and r , using the given volume of the bin. [1]

d. Show that the total external surface area, A , of the bin is $A = \pi r^2 + \frac{16000}{r}$. [2]

e. Write down $\frac{\mathrm{d}A}{\mathrm{d}r}$. [3]

f. (i) Find the value of r that minimizes the total external surface area of the wastepaper bin. [5]

(ii) Calculate the value of h corresponding to this value of r .

g. Determine whether Merryn's design is an improvement upon Nadia's. Give a reason. [2]

Markscheme

a. (i) $\text{Area} = \pi(5)^2$ ***(M1)***

$= 78.5\ (\text{cm}^2)$ (78.5398...) ***(A1)(G2)***

Note: Accept 25π .

(ii) $8000 = 78.5398\ldots \times h$ ***(M1)***
$h = 102\ (\text{cm})$ (101.859...) ***(A1)*(ft)*(G2)***

Note: Follow through from their answer to part (a)(i).

(iii) $\text{Area} = \pi(5)^2 + 2\pi(5)(101.859\ldots)$ ***(M1)(M1)***

Note: Award ***(M1)*** for their substitution in curved surface area formula, ***(M1)*** for addition of their two areas.

$= 3280\ (\text{cm}^2)$ (3278.53...) ***(A1)*(ft)*(G2)***

Note: Follow through from their answers to parts (a)(i) and (ii).

b. No, it is too tall/narrow. ***(A1)*(ft)*(R1)***

Note: Follow through from their value for h.

c. $8000 = \pi r^2 h$ ***(A1)***

d. $A = \pi r^2 + 2\pi r\left(\frac{8000}{\pi r^2}\right)$ ***(A1)(M1)***

Note: Award ***(A1)*** for correct rearrangement of **their** part (c), ***(M1)*** for substitution of **their** rearrangement into area formula.

$= \pi r^2 + \frac{16000}{r}$ ***(AG)***

e. $\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r - 16000r^{-2}$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for $2\pi r$, ***(A1)*** for -16000 ***(A1)*** for r^{-2} . If an extra term is present award at most ***(A1)(A1)(A0)***.

f. (i) $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ ***(M1)***

$2\pi r^3 - 16000 = 0$ ***(M1)***

$r = 13.7 \text{ cm } (13.6556\ldots)$ ***(A1)*(ft)**

Note: Follow through from their part (e).

(ii) $h = \frac{8000}{\pi(13.65\ldots)^2}$ ***(M1)***

$= 13.7 \text{ cm } (13.6556\ldots)$ ***(A1)*(ft)**

Note: Accept 13.6 if 13.7 used.

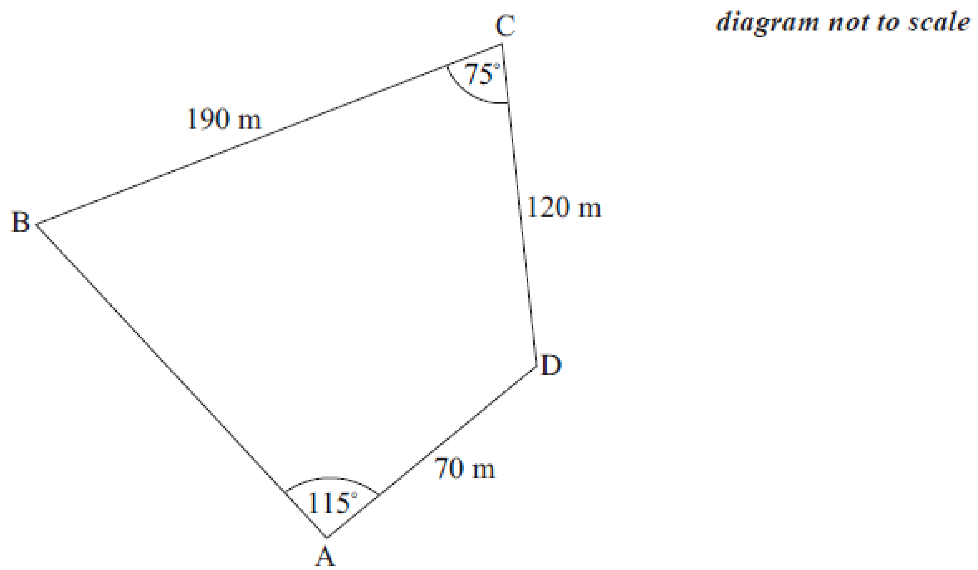
g. Yes or No, accompanied by a consistent and sensible reason. ***(A1)(R1)***

Note: Award ***(A0)(R0)*** if no reason is given.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]

Pauline owns a piece of land ABCD in the shape of a quadrilateral. The length of BC is 190 m , the length of CD is 120 m , the length of AD is 70 m , the size of angle BCD is 75° and the size of angle BAD is 115° .



Pauline decides to sell the triangular portion of land ABD . She first builds a straight fence from B to D .

a. Calculate the length of the fence. [3]

b. The fence costs 17 USD per metre to build. [2]

Calculate the cost of building the fence. Give your answer correct to the nearest USD.

c. Show that the size of angle ABD is 18.8° , correct to three significant figures. [3]

d. Calculate the area of triangle ABD . [4]

e. She sells the land for 120 USD per square metre. [2]

Calculate the value of the land that Pauline sells. Give your answer correct to the nearest USD.

f. Pauline invests 300000 USD from the sale of the land in a bank that pays compound interest compounded annually. [4]

Find the interest rate that the bank pays so that the investment will double in value in 15 years.

Markscheme

a. $\mathrm{BD}^2 = 190^2 + 120^2 - 2(190)(120)\cos 75^\circ$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted cosine formula, ***(A1)*** for correct substitution.

$= 197$ m ***(A1)(G2)***

Note: If radians are used award a maximum of ***(M1)(A1)(A0)***.

[3 marks]

b. $\text{cost} = 196.717\ldots \times 17$ ***(M1)***

$= 3344 \text{ USD}$ ***(A1)*(ft)*(G2)***

Note: Accept 3349 from 197.

[2 marks]

c. $\frac{\sin(\mathrm{ABD})}{70} = \frac{\sin(115^\circ)}{196.7}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted sine formula, ***(A1)*** for correct substitution.

$= 18.81^\circ\ldots$ ***(A1)*(ft)**
$= 18.8^\circ$ ***(AG)***

Notes: Both the unrounded and rounded answers must be seen for the final ***(A1)*** to be awarded. Follow through from their (a). If 197 is used the unrounded answer is $= 18.78^\circ\ldots$

[3 marks]

d. $\text{angle BDA} = 46.2^\circ$ ***(A1)***

$\text{Area} = \frac{70\times(196.717\ldots)\times\sin(46.2^\circ)}{2}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituted area formula, ***(A1)*** for correct substitution.

$\text{Area ABD} = 4970\ \text{m}^2$ ***(A1)*(ft)*(G2)***

Notes: If 197 used answer is 4980.

Notes: Follow through from (a) only. Award ***(G2)*** if there is no working shown and 46.2° not seen. If 46.2° seen without subsequent working, award ***(A1)(G2)***.

[4 marks]

e. $4969.38\ldots \times 120$ ***(M1)***

$= 596327\ \text{USD}$ ***(A1)*(ft)*(G2)***

Notes: Follow through from their (d).

[2 marks]

f. $300000\left(1+\frac{r}{100}\right)^{15} = 600000$ or equivalent ***(A1)(M1)(A1)***

Notes: Award ***(A1)*** for 600000 seen or implied by alternative formula, ***(M1)*** for substituted CI formula, ***(A1)*** for correct substitutions.

$r = 4.73$ ***(A1)*(ft)*(G3)***

Notes: Award ***(G3)*** for 4.73 with no working. Award ***(G2)*** for 4.7 with no working.

[4 marks]

Examiners report

a. Most candidates were able to recognise cosine rule, and substitute correctly. Where the final answer was not attained, this was mainly due to further unnecessary manipulation; the GDC should be used efficiently in such a case. Some students used the answer given and sine rule – this gained no credit.

b. Most candidates were able to recognise cosine rule, and substitute correctly. Where the final answer was not attained, this was mainly due to further unnecessary manipulation; the GDC should be used efficiently in such a case. Some students used the answer given and sine rule – this gained no credit.

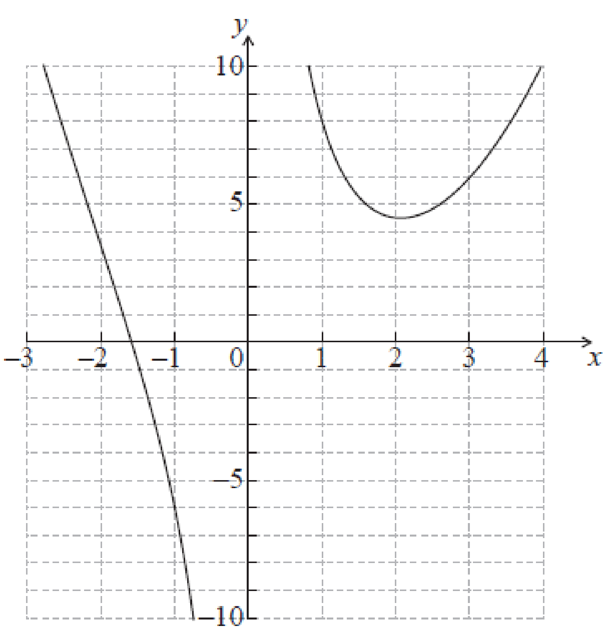
c. Most candidates were able to recognise cosine rule, and substitute correctly. Where the final answer was not attained, this was mainly due to further unnecessary manipulation; the GDC should be used efficiently in such a case. Some students used the answer given and sine rule – this gained no credit.

d. Again, most candidates used the appropriate area formula – however, some did not appreciate the purpose of the given answer and were unable to complete the question accurately.

e. Again, most candidates used the appropriate area formula – however, some did not appreciate the purpose of the given answer and were unable to complete the question accurately.

f. The final part, in which compound interest was again asked for, tested most candidates but there were many successful attempts using either the GDC's finance package or correct use of the formula. Care must be taken with the former to show some indication of the values to be used in the context of the question. With the latter approach marks were again lost due to a lack of appreciation of the difference between interest and value.

The diagram shows part of the graph of $f(x) = x^2 - 2x + \frac{9}{x}$, where $x \neq 0$.



a. Write down [5]

(i) the equation of the vertical asymptote to the graph of $y = f(x)$;

(ii) the solution to the equation $f(x) = 0$;

(iii) the coordinates of the local minimum point.

b. Find $f'(x)$. [4]

c. Show that $f'(x)$ can be written as $f'(x) = \frac{2x^3-2x^2-9}{x^2}$. [2]

d. Find the gradient of the tangent to $y = f(x)$ at the point $\mathrm{A}(1, 8)$. [2]

e. The line, L, passes through the point A and is perpendicular to the tangent at A. [1]

Write down the gradient of L .

f. The line, L , passes through the point A and is perpendicular to the tangent at A. [3]

Find the equation of L . Give your answer in the form $y = mx + c$.

g. The line, L , passes through the point A and is perpendicular to the tangent at A. [2]

L also intersects the graph of $y = f(x)$ at points B and C . Write down the **x-coordinate** of B and of C .

Markscheme

a. (i) $x = 0$ ***(A1)(A1)***

Note: Award ***(A1)*** for $x =$ a constant, ***(A1)*** for the constant in their equation being 0.

(ii) $-1.58\,(-1.58454\ldots)$ ***(G1)***

Note: Accept -1.6, do not accept -2 or -1.59.

(iii) $(2.06, 4.49)\ (2.06020\ldots, 4.49253\ldots)$ ***(G1)(G1)***

Note: Award at most ***(G1)(G0)*** if brackets not used. Award ***(G0)(G1)*****(ft)** if coordinates are reversed.

Note: Accept $x = 2.06$, $y = 4.49$.

Note: Accept 2.1, do not accept 2.0 or 2. Accept 4.5, do not accept 5 or 4.50.

[5 marks]

b. $f'(x) = 2x - 2 - \frac{9}{x^2}$ ***(A1)(A1)(A1)(A1)***

Notes: Award ***(A1)*** for $2x$, ***(A1)*** for -2, ***(A1)*** for -9, ***(A1)*** for x^{-2} . Award a maximum of ***(A1)(A1)(A1)(A0)*** if there are extra terms present.

[4 marks]

c. $f'(x) = \frac{x^2(2x-2)}{x^2} - \frac{9}{x^2}$ ***(M1)***

Note: Award ***(M1)*** for taking the correct common denominator.

$= \frac{(2x^3-2x^2)}{x^2} - \frac{9}{x^2}$ ***(M1)***

Note: Award ***(M1)*** for multiplying brackets or equivalent.

$= \frac{2x^3-2x^2-9}{x^2}$ ***(AG)***

Note: The final ***(M1)*** is not awarded if the given answer is not seen.

[2 marks]

d. $f'(1) = \frac{2(1)^3-2(1)-9}{(1)^2}$ ***(M1)***

$= -9$ ***(A1)(G2)***

Note: Award ***(M1)*** for substitution into **given** (or their correct) $f'(x)$. There is no follow through for use of their incorrect derivative.

[2 marks]

e. $\frac{1}{9}$ ***(A1)*(ft)**

Note: Follow through from part (d).

[1 mark]

f. $y - 8 = \frac{1}{9}(x - 1)$ ***(M1)(M1)***

Notes: Award ***(M1)*** for substitution of their gradient from (e), ***(M1)*** for substitution of given point. Accept all forms of straight line.

$y = \frac{1}{9}x + \frac{71}{9}$ $(y = 0.111111\ldots x + 7.88888\ldots)$ ***(A1)*(ft)*(G3)***

Note: Award the final ***(A1)*(ft)** for a correctly rearranged formula of **their** straight line in (f). Accept $0.11x$, do not accept $0.1x$. Accept 7.9, do not accept 7.88, do not accept 7.8.

[3 marks]

g. $-2.50, 3.61$ $(-2.49545\ldots, 3.60656\ldots)$ ***(A1)*(ft)*(A1)*(ft)**

Notes: Follow through from their line L from part (f) even if no working shown. Award at most ***(A0)(A1)*(ft)** if their correct coordinate pairs given.

Note: Accept -2.5, do not accept -2.49. Accept 3.6, do not accept 3.60.

[2 marks]

Examiners report

a. As usual, the content in this question caused difficulty for many candidates. However, for those with a sound grasp of the topic, there were many very successful attempts. The curve was given so that a comparison could be made to a GDC version and the correct form of the derivative was also given to permit weaker candidates to progress to the latter stages. Unfortunately, some decided to proceed with their own incorrect versions, in which case **very limited follow through accrued**. It should be emphasized to candidates that when an answer is given in this way it should be used in subsequent parts of the question.

As in previous years, much of the question could have been answered successfully by using the GDC. However, it was also clear that a large number of candidates did not attempt either to verify their work with their GDC or to use it in place of an algebraic approach.

Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested. Some centres still do not teach the differential calculus.

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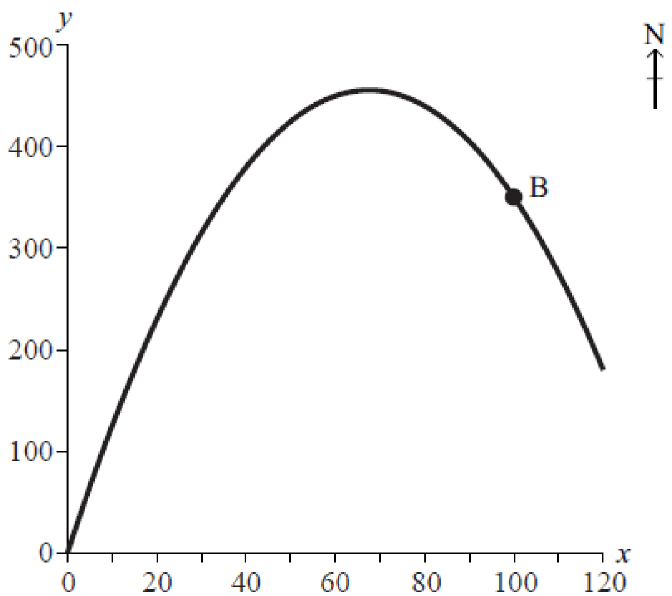
The diagram shows an **aerial** view of a bicycle track. The track can be modelled by the quadratic function

$y = \frac{-x^2}{10} + \frac{27}{2}x$, where $x \geqslant 0,\ y \geqslant 0$

(x , y) are the coordinates of a point x metres east and y metres north of O , where O is the origin (0, 0) . B is a point on the bicycle track with coordinates (100, 350) .



a. The coordinates of point A are (75, 450). Determine whether point A is on the bicycle track. Give a reason for your answer. [3]

b. Find the derivative of $y = \frac{-x^2}{10} + \frac{27}{2}x$. [2]

c. Use the answer in part (b) to determine if A (75, 450) is the point furthest north on the track between O and B. Give a reason for your answer. [4]

d. (i) Write down the midpoint of the line segment OB. [3]

(ii) Find the gradient of the line segment OB.

e. Scott starts from a point C(0,150) . He hikes along a straight road towards the bicycle track, parallel to the line segment OB. [3]

Find the equation of Scott's road. Express your answer in the form $ax + by = c$, where a, b and $c \in \mathbb{R}$.

f. Use your graphic display calculator to find the coordinates of the point where Scott first crosses the bicycle track. [2]

Markscheme

a. $y = -\frac{75^2}{10} + \frac{27}{2} \times 75$ ***(M1)***

Note: Award ***(M1)*** for substitution of 75 in the formula of the function.

= 450 ***(A1)***

Yes, point A is on the bike track. ***(A1)***

Note: Do not award the final ***(A1)*** if correct working is not seen.

b. $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2x}{10} + \frac{27}{2} \left(\frac{\mathrm{d}y}{\mathrm{d}x} = -0.2x + 13.5 \right)$ ***(A1)(A1)***

Notes: Award ***(A1)*** for each correct term. If extra terms are seen award at most ***(A1)(A0)***. Accept equivalent forms.

c. $-\frac{2x}{10} + \frac{27}{2} = 0$ ***(M1)***

Note: Award ***(M1)*** for equating their derivative from part (b) to zero.

$x = 67.5$ ***(A1)*****(ft)**

Note: Follow through from their derivative from part (b).

$(\text{Their})\ 67.5 \neq 75$ ***(R1)***

Note: Award ***(R1)*** for a comparison of their 67.5 with 75. Comparison may be implied (*eg* 67.5 is the *x*-coordinate of the furthest north point).

OR

$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2 \times (75)}{10} + \frac{27}{2}$ ***(M1)***

Note: Award ***(M1)*** for substitution of 75 into their derivative from part (b).

$= -1.5$ ***(A1)*****(ft)**

Note: Follow through from their derivative from part (b).

$(\text{Their}) - 1.5 \neq 0$ ***(R1)***

Note: Award ***(R1)*** for a comparison of their –1.5 with 0. Comparison may be implied (*eg* The gradient of the parabola at the furthest north point (vertex) is 0).

Hence A is not the furthest north point. ***(A1)*****(ft)**

Note: Do not award ***(R0)(A1)*****(ft)**. Follow through from their derivative from part (b).

d. (i) M(50,175) ***(A1)***

Note: If parentheses are omitted award ***(A0)***. Accept *x* = 50, *y* = 175.

(ii) $\frac{350-0}{100-0}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in gradient formula.

$= 3.5\left(\frac{350}{100}, \frac{7}{2}\right)$ ***(A1)*(ft)*(G2)***

Note: Follow through from (d)(i) if midpoint is used to calculate gradient. Award ***(G1)(G0)*** for answer 3.5*x* without working.

e. $y = 3.5x + 150$ ***(A1)*(ft)*(A1)*(ft)**

Note: Award ***(A1)*(ft)** for using their gradient from part (d), ***(A1)*(ft)** for correct equation of line.

$3.5x - y = -150$ **or** $7x - 2y = -300$ (or equivalent) ***(A1)*(ft)**

Note: Award ***(A1)*(ft)** for expressing their equation in the form $ax + by = c$.

f. (18.4, 214) (18.3772..., 214.320...) ***(A1)*(ft)*(A1)*(ft)*(G2)*(ft)**

Notes: Follow through from their equation in (e). Coordinates must be positive for follow through marks to be awarded. If parentheses are omitted and not already penalized in (d)(i) award at most ***(A0)(A1)*(ft)**. If coordinates of the two intersection points are given award ***(A0)(A1)*(ft)**. Accept *x* = 18.4, *y* = 214.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]

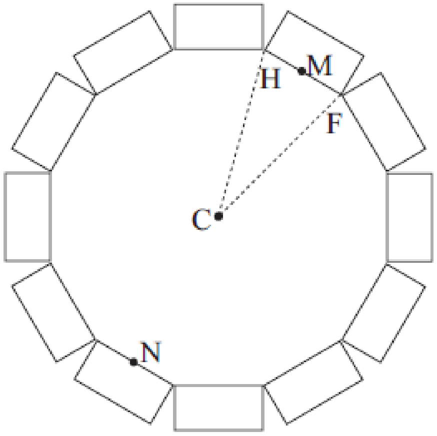
A gardener has to pave a rectangular area 15.4 metres long and 5.5 metres wide using rectangular bricks. The bricks are 22 cm long and 11 cm wide.

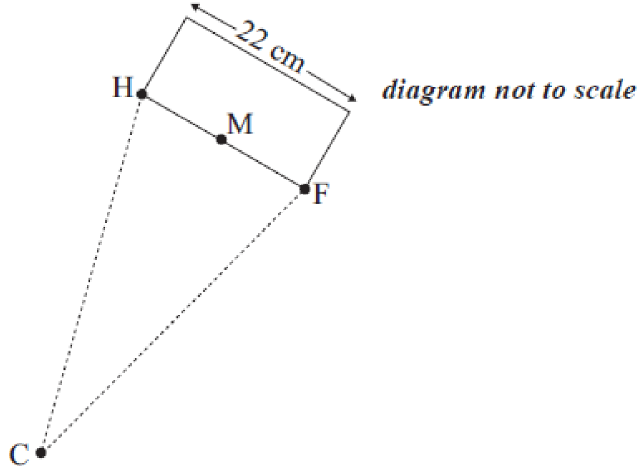
The gardener decides to have a triangular lawn ABC, instead of paving, in the middle of the rectangular area, as shown in the diagram below.

onbekend.png

The distance AB is 4 metres, AC is 6 metres and angle BAC is 40°.

In another garden, twelve of the same rectangular bricks are to be used to make an edge around a small garden bed as shown in the diagrams below. FH is the length of a brick and C is the centre of the garden bed. M and N are the midpoints of the long edges of the bricks on opposite sides of the garden bed.





The garden bed has an area of 5419 cm^2. It is covered with soil to a depth of 2.5 cm.

It is estimated that 1 kilogram of soil occupies 514 cm^3.

a.i. Calculate the total area to be paved. Give your answer in cm^2. [3]

a.ii. Write down the area of each brick. [1]

a.iii. Find how many bricks are required to pave the total area. [2]

b.i. Find the length of BC. [3]

b.ii. Hence write down the perimeter of the triangular lawn. [1]

b.iii. Calculate the area of the lawn. [2]

b.iv. Find the percentage of the rectangular area which is to be lawn. [3]

c.i. Find the angle FCH. [2]

c.ii. Calculate the distance MN from one side of the garden bed to the other, passing through C. [3]

d. Find the volume of soil used. [2]

e. Find the number of kilograms of soil required for this garden bed. [2]

Markscheme

a.i. 15.4×5.5 ***(M1)***

84.7 m^2 ***(A1)***

$= 847000$ cm^2 ***(A1)(G3)***

Note: Award ***(G2)*** if 84.7 m^2 seen with no working.

OR

1540 × 550 ***(A1)(M1)***

= 847000 cm^2 ***(A1)*(ft)*(G3)***

Note: Award ***(A1)*** for both dimensions converted correctly to cm, ***(M1)*** for multiplication of both dimensions. ***(A1)*(ft)** for correct product of their sides in cm.

[3 marks]

a.ii.242 cm^2 (0.0242 m^2) ***(A1)***

[1 marks}

a.iii.$\frac{15.4}{0.22} = 70$ ***(M1)***

$\frac{5.5}{0.11} = 50$

$70 \times 50 = 3500$ ***(A1)(G2)***

OR

$\frac{847000}{242} = 3500$ ***(M1)(A1)*(ft)*(G2)***

Note: Follow through from parts (a) (i) and (ii).

[2 marks]

b.i.$\mathrm{BC}^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \times \cos 40°$ ***(M1)(A1)***

$\mathrm{BC} = 3.90\ \mathrm{m}$ ***(A1)(G2)***

Note: Award ***(M1)*** for correct substituted formula, ***(A1)*** for correct substitutions, ***(A1)*** for correct answer.

[3 marks]

b.ii.perimeter = 13.9 m ***(A1)*(ft)*(G1)***

Notes: Follow through from part (b) (i).

[1 mark]

b.iii.

$\text{Area} = \frac{1}{2} \times 4 \times 6 \times \sin 40°$ ***(M1)***

= 7.71 m^2 ***(A1)*(ft)*(G2)***

Notes: Award ***(M1)*** for correct formula and correct substitution, ***(A1)*(ft)** for correct answer.

[2 marks]

b.iv. $\frac{7.713}{84.7} \times 100\ \% = 9.11\ \%$ ***(A1)(M1)(A1)*(ft)*(G2)***

Notes: Accept 9.10 %.

Award ***(A1)*** for both measurements correctly written in the same unit, ***(M1)*** for correct method, ***(A1)*(ft)** for correct answer.

Follow through from (b) (iii) and from consistent error in conversion of units throughout the question.

[3 marks]

c.i. $\frac{360^\circ}{12}$ ***(M1)***

$= 30^\circ$ ***(A1)(G2)***

[2 marks]

c.ii. $\mathrm{MN} = 2 \times \frac{11}{\tan 15}$ ***(A1)*(ft)*(M1)***

OR

$\mathrm{MN} = 2 \times 11 \tan 75^\circ$

$\mathrm{MN} = 82.1\ \mathrm{cm}$ ***(A1)*(ft)*(G2)***

Notes: Award ***(A1)*** for 11 and 2 seen (implied by 22 seen), ***(M1)*** for dividing by tan15 (or multiplying by tan 75).

Follow through from their angle in part (c) (i).

[3 marks]

d. volume = 5419 × 2.5 ***(M1)***

= 13500 cm^3 ***(A1)(G2)***

[2 marks]

e. $\frac{13547.34\ldots}{514} = 26.4$ ***(M1)(A1)*(ft)*(G2)***

Note: Award ***(M1)*** for dividing their part (d) by 514.

Accept 26.3.

[2 marks]

Examiners report

a.i. Part (a) was well done except for the fact that very few students were able to convert correctly from m^2 to cm^2 and this was very disappointing.

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a.iii. Part (a) was well done except for the fact that very few students were able to convert correctly from m^2 to m^2 and this was very disappointing.

b.i. In part (b) the cosine rule and the area of a triangle were well done. In some cases units were missing and therefore a unit penalty was applied.

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b.iv. In part (b) the cosine rule and the area of a triangle were well done. In some cases units were missing and therefore a unit penalty was applied.

c.i. Part (c) was clearly the most difficult one for the students. The general impression was that they did not read the diagram in detail. A number of candidates could not distinguish the circle from the triangle and hence used an incorrect method to find the radius.

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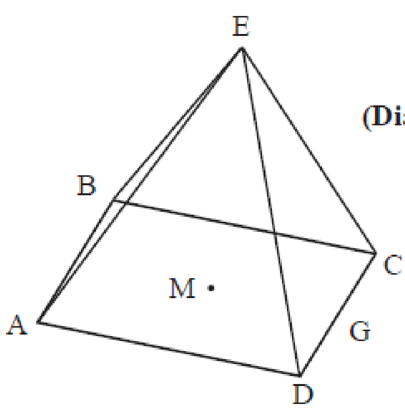
d. It was pleasing to see candidates recovering well to get full marks for the last two parts.

e. It was pleasing to see candidates recovering well to get full marks for the last two parts.

Mal is shopping for a school trip. He buys 50 tins of beans and 20 packets of cereal. The total cost is 260 Australian dollars (AUD).

The triangular faces of a square based pyramid, ABCDE, are all inclined at 70° to the base. The edges of the base ABCD are all 10 cm and M is the centre. G is the mid-point of CD.



(Diagram not to scale)

i.a. Write down an equation showing this information, taking b to be the cost of one tin of beans and c to be the cost of one packet of cereal in AUD. [1]

i.b. Stephen thinks that Mal has not bought enough so he buys 12 more tins of beans and 6 more packets of cereal. He pays 66 AUD. [1]

Write down another equation to represent this information.

i.c. Stephen thinks that Mal has not bought enough so he buys 12 more tins of beans and 6 more packets of cereal. He pays 66 AUD. [2]

Find the cost of one tin of beans.

i.d. (i) Sketch the graphs of the two equations from parts (a) and (b). [4]

(ii) Write down the coordinates of the point of intersection of the two graphs.

ii.a. Using the letters on the diagram draw a triangle showing the position of a 70° angle. [1]

ii.b. Show that the height of the pyramid is $13.7\ \text{cm}$, to 3 significant figures. [2]

ii.c. Calculate [4]

(i) the length of EG;

(ii) the size of angle DEC.

ii.d. Find the total surface area of the pyramid. [2]

ii.e. Find the volume of the pyramid. [2]

Markscheme

i.a. $50b + 20c = 260$ ***(A1)***

[1 mark]

i.b. $12b + 6c = 66$ ***(A1)***

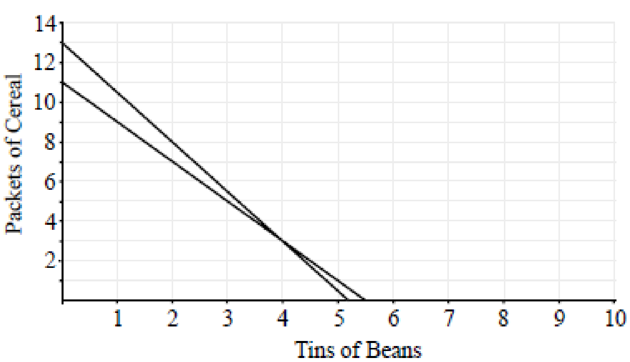
[1 mark]

i.c. Solve to get $b = 4$ ***(M1)(A1)*****(ft)*****(G2)***

Note: ***(M1)*** for attempting to solve the equations simultaneously.

[2 marks]

i.d. (i)



(A1)(A1)(A1)

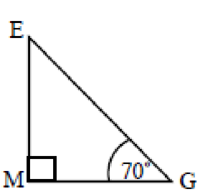
Notes: Award ***(A1)*** for labels and some idea of scale, ***(A1)*****(ft)*****(A1)*****(ft)** for each line.
The axis can be reversed.

(ii) $(4, 3)$ or $(3, 4)$ ***(A1)*****(ft)**

Note: Accept $b = 4$, $c = 3$

[4 marks]

ii.a.



(A1)

[1 mark]

ii.b. $\tan 70 = \frac{h}{5}$ ***(M1)***

$h = 5\tan 70 = 13.74$ ***(A1)***

$h = 13.7 \text{ cm}$ ***(AG)***

[2 marks]

ii.c. *Unit penalty **(UP)** is applicable in this part of the question where indicated in the left hand column.*

(i) $\text{EG}^2 = 5^2 + 13.7^2$ OR $5^2 + (5\tan 70)^2$ ***(M1)***

(UP) $\text{EG} = 14.6 \text{ cm}$ ***(A1)(G2)***

(ii) $\text{DEC} = 2 \times \tan^{-1}\left(\frac{5}{14.6}\right)$ ***(M1)***

$= 37.8^\circ$ ***(A1)*****(ft)*****(G2)***

[4 marks]

ii.d. *Unit penalty **(UP)** is applicable in this part of the question where indicated in the left hand column.*

$\text{Area} = 10 \times 10 + 4 \times 0.5 \times 10 \times 14.619$ ***(M1)***

(UP) $= 392 \text{ cm}^2$ ***(A1)*****(ft)*****(G2)***

[2 marks]

ii.e. *Unit penalty **(UP)** is applicable in this part of the question where indicated in the left hand column.*

$\text{Volume} = \frac{1}{3} \times 10 \times 10 \times 13.7$ ***(M1)***

(UP) $= 457 \text{ cm}^3$ (458 cm^3) ***(A1)(G2)***

[2 marks]

Examiners report

i.a. Most candidates managed to write down the equation.

i.b. Most candidates managed to write down the equation.

i.c. Many managed to find the correct answer and the others tried their best but made some mistake in the process.

i.d. (i) Few candidates sketched the graphs well. Few used a ruler.

(ii) Many candidates could not be awarded ft from their graph because the answer they gave was not possible.

ii.a. Very few correct drawings.

ii.b. Some managed to show this more by good fortune and ignoring their original triangle than by good reasoning.

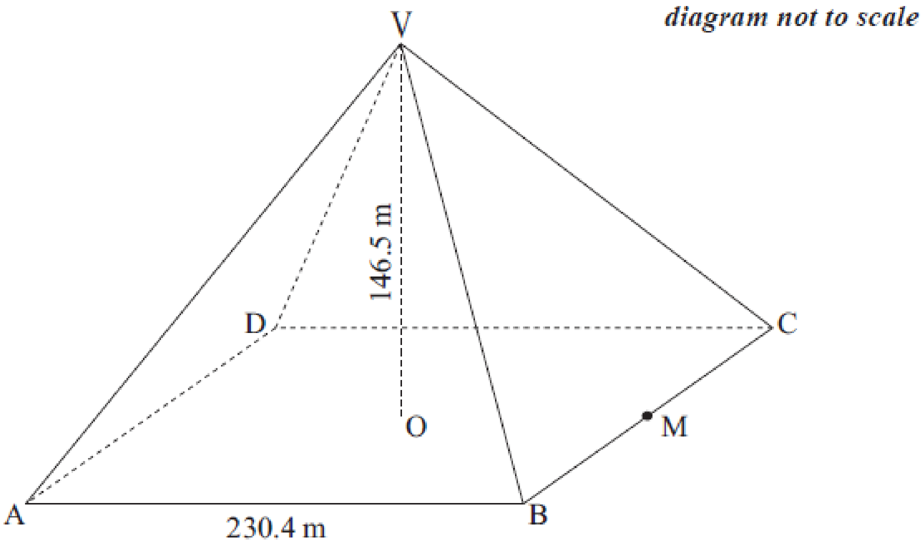
ii.c. (i) Many found this as ft from the previous part. Some lost a UP here.

(ii) This was not well done. The most common answer was 40°.

ii.d. Many managed this or were awarded ft points.

ii.e. This was well done and most candidates also remembered their units on this part.

The Great Pyramid of Cheops in Egypt is a square based pyramid. The base of the pyramid is a square of side length 230.4 m and the vertical height is 146.5 m. The Great Pyramid is represented in the diagram below as ABCDV . The vertex V is directly above the centre O of the base. M is the midpoint of BC.



a. (i) Write down the length of OM . [3]

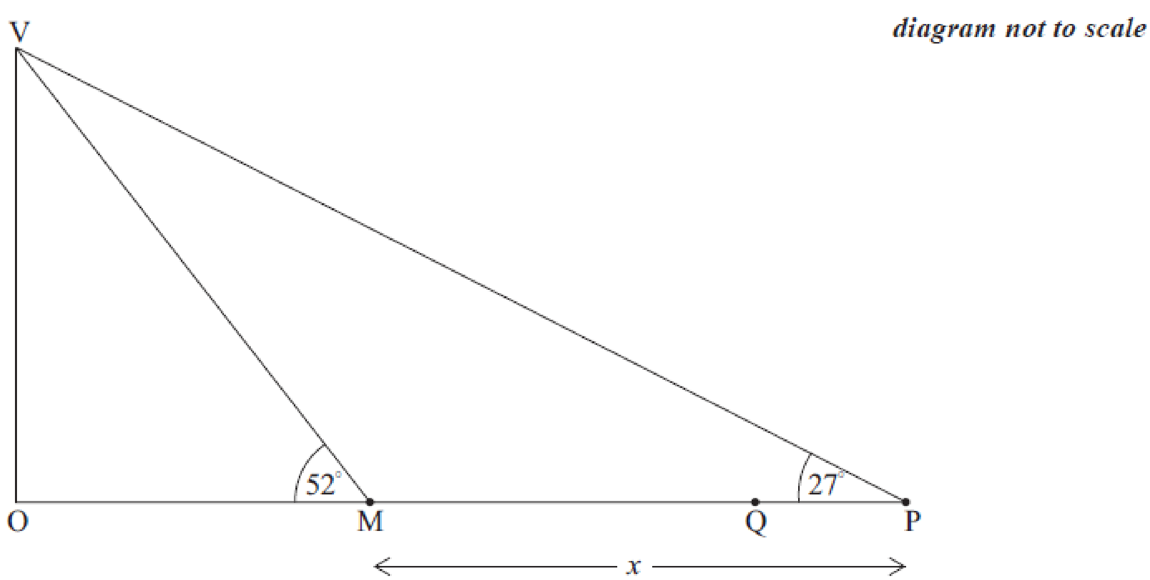
(ii) Find the length of VM .

b. Find the area of triangle VBC . [2]

c. Calculate the volume of the pyramid. [2]

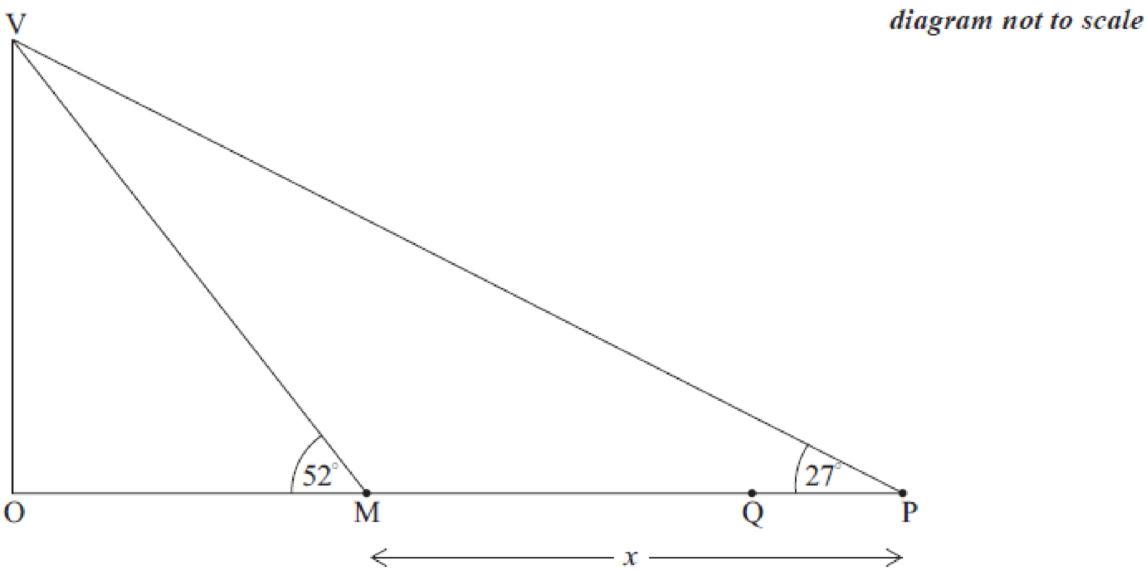
d. Show that the angle between the line VM and the base of the pyramid is 52° correct to 2 significant figures. [2]

e. Ahmed is at point P , a distance *x* metres from M on horizontal ground, as shown in the following diagram. The size of angle VPM is 27° . Q is a point on MP . [1]



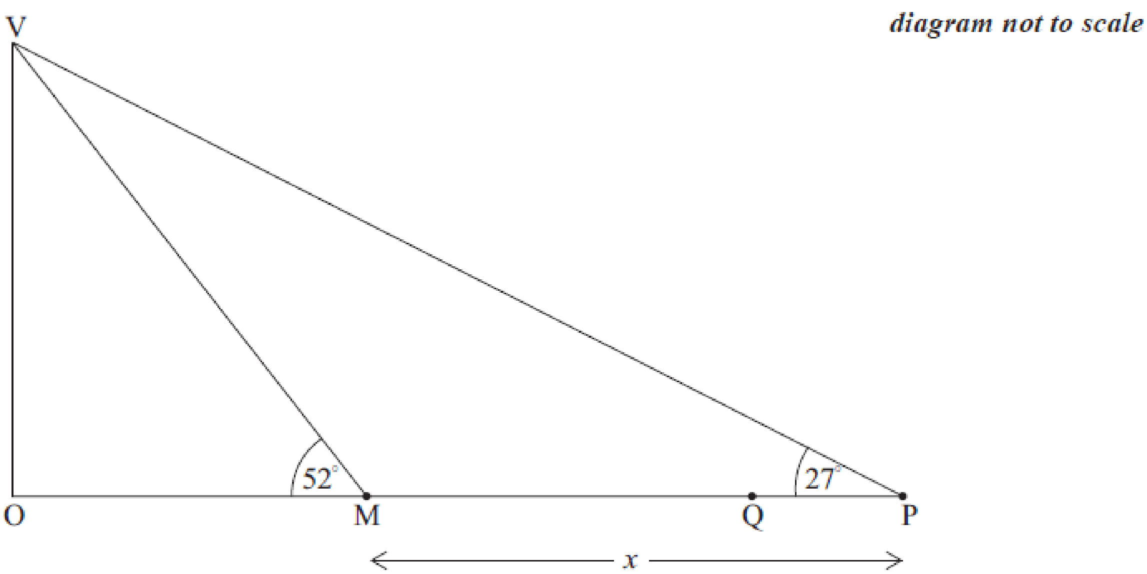
Write down the size of angle VMP .

f. Ahmed is at point P , a distance *x* metres from M on horizontal ground, as shown in the following diagram. The size of angle VPM is 27° . Q is a point on MP . [4]



Using your value of VM from part (a)(ii), find the value of *x*.

g. Ahmed is at point P , a distance *x* metres from M on horizontal ground, as shown in the following diagram. The size of angle VPM is 27° . Q is a point on MP . [4]



Ahmed walks 50 m from P to Q.

Find the length of QV, the distance from Ahmed to the vertex of the pyramid.

Markscheme

a. (i) 115.2 (m) ***(A1)***

Note: Accept 115 (m)

(ii) $\sqrt{(146.5^2 + 115.2^2)}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution.

186 (m) (186.368…) ***(A1)*(ft)*(G2)***

Note: Follow through from part (a)(i).

[3 marks]

b. $\frac{1}{2} \times 230.4 \times 186.368...$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in area of the triangle formula.

21500 m^2 (21469.6…m^2) ***(A1)*(ft)*(G2)***

Notes: The final answer is 21500 m^2; **units are required**. Accept 21400 m^2 for use of 186 m and/or 115 m.

[2 marks]

c. $\frac{1}{3} \times 230.4^2 \times 146.5$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in volume formula.

2590000 m^3 (2592276.48 m^3) ***(A1)(G2)***

Note: The final answer is 2590000 m^3; **units are required** but do not penalise missing or incorrect units if this has already been penalised in part (b).

[2 marks]

d. $\tan^{-1}\left(\frac{146.5}{115.2}\right)$ ***(M1)***

Notes: Award ***(M1)*** for correct substituted trig ratio. Accept alternate correct trig ratios.

= 51.8203...= 52° ***(A1)(AG)***

Notes: Both the unrounded answer and the final answer must be seen for the ***(A1)*** to be awarded. Accept 51.96° = 52°, 51.9° = 52° or 51.7° = 52°

e. 128° ***(A1)***

[1 mark]

f. $\frac{186.368}{\sin 27} = \frac{x}{\sin 25}$ ***(A1)(M1)(A1)*(ft)**

Notes: Award ***(A1)*(ft)** for their angle MVP seen, follow through from their part (e). Award ***(M1)*** for substitution into sine formula, ***(A1)*** for correct substitutions. Follow through from their VM and their angle VMP.

x = 173 (m) (173.490...) ***(A1)*(ft)*(G3)***

Note: Accept 174 from use of 186.4.

[4 marks]

g. VQ^2 = $(186.368...)^2 + (123.490...)^2 - 2 \times (186.368...) \times (123.490...) \times \cos 128$ ***(A1)*(ft)*(M1)(A1)*(ft)**

Notes: Award ***(A1)*(ft)** for 123.490...(123) seen, follow through from their x (PM) in part (f), ***(M1)*** for substitution into cosine formula, ***(A1)*(ft)** for correct substitutions. Follow through from their VM and their angle VMP.

OR

173.490... − 50 = 123.490... (123) ***(A1)*(ft)**

115.2 + 123.490... = 238.690... ***(A1)*(ft)**

$\mathrm{VQ} = \sqrt{(146.5^2 + 238.690...^2)}$ ***(M1)***

VQ = 280 (m) (280.062...) ***(A1)*(ft)*(G3)***

Note: Accept 279 (m) from use of 3 significant figure answers.

[4 marks]

Examiners report

a. (a) This part was very well done on the whole.

b. (b) Amazingly badly done. Many candidates used 146.4 for the height and others tried unsuccessfully to find slant heights and angles to that they could use the area of a triangle formula $\frac{1}{2}ab\sin C$.

c. (c) This was fairly well done.

d. (d) Quite a few candidates managed to show this although they did not always put down the unrounded answer and so lost the last mark. Some even tried to use 52° to verify its value.

.

e. (e) Very well done on the whole – even if part (d) was wrong.

f. (f) This was well done by those who attempted it. Not all candidates used VM to find x and so lost one mark. There were quite a few different methods of finding the answer.

g. (g) Again this was well done by those who attempted it. Again there were many different ways to reach the correct answer.

The diagram shows a Ferris wheel that moves with constant speed and completes a rotation every 40 seconds. The wheel has a radius of 12 m and its lowest point is 2 m above the ground.

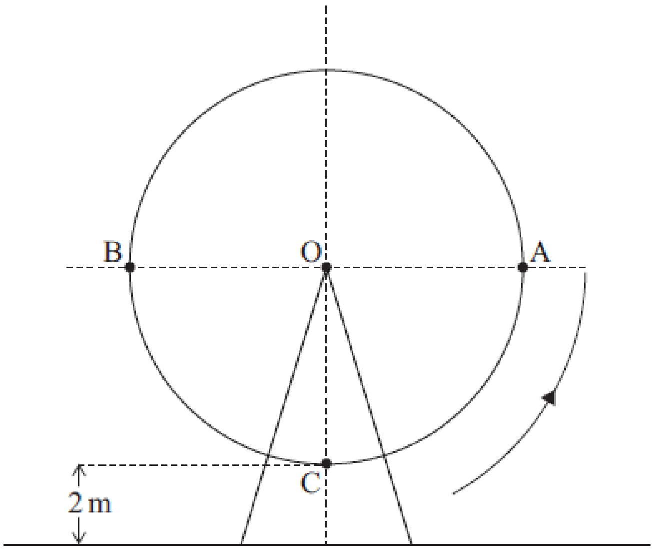


diagram not to scale

a. Initially, a seat C is vertically below the centre of the wheel, O. It then rotates in an anticlockwise (counterclockwise) direction. [2]

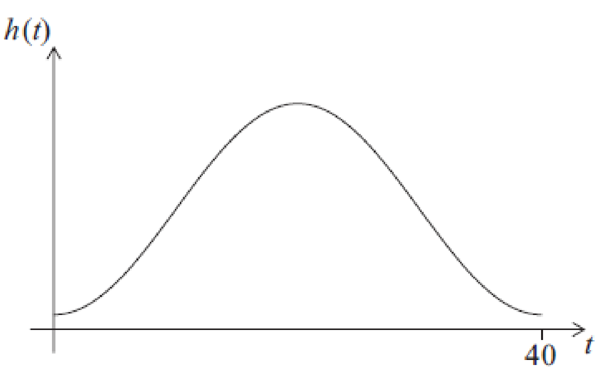
Write down

(i) the height of O above the ground;

(ii) the maximum height above the ground reached by C .

b. In a revolution, C reaches points A and B , which are at the same height above the ground as the centre of the wheel. Write down the number of seconds taken for C to first reach A and then B . [2]

c. The sketch below shows the graph of the function, $h(t)$, for the height above ground of C, where h is measured in metres and t is the time in seconds, $0 \leqslant t \leqslant 40$. [4]



Copy the sketch and show the results of part (a) and part (b) on your diagram. Label the points clearly with their coordinates.

Markscheme

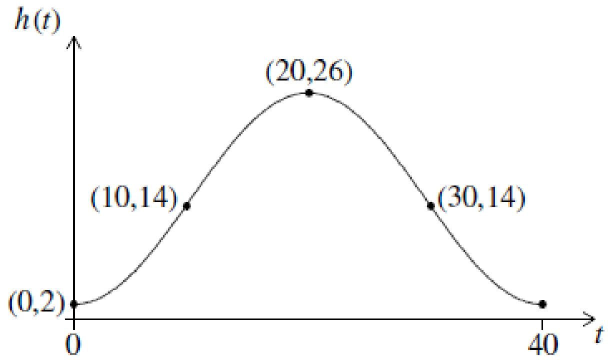
a. (i) 14 m ***(A1)***

(ii) 26 m ***(A1)***

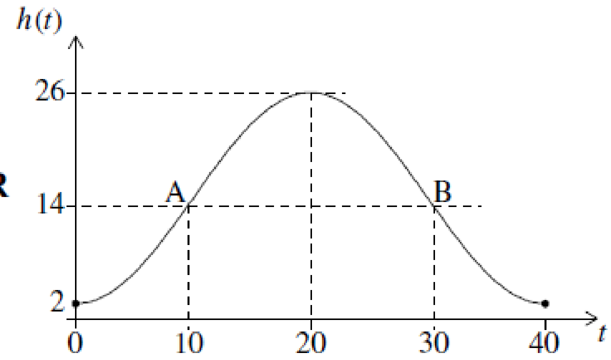
[2 marks]

b. A:10, B:30 ***(A1)(A1)***

[2 marks]

c. 

OR



(A1)*(ft)*(A1)*(ft)*(A1)

(ft)*(A1)*(ft)

Note: Award ***(A1)*(ft)** for coordinates of each point clearly indicated either by scale or by coordinate pairs. Points need not be labelled A and B in the second diagram. Award a maximum of ***(A1)(A0)(A1)*(ft)*(A1)*(ft)** if coordinates are reversed. Do not penalise reversed coordinates if this has already been penalised in Q4(a)(iii).

[4 marks]

Examiners report

a. Most candidates were able to start this question. Those of an average ability completed it to the end of part (c) and the best gained good success in the latter parts. Its purpose was to discriminate at the highest level and this it did.

Some concerns were raised on the G2 forms as to the appropriateness of this question. However, the MSSL course tries in part to link areas of the syllabus to "real-life" situations and address these. A look back to past years' examination papers, and to the syllabus documentation, should yield similar examples.

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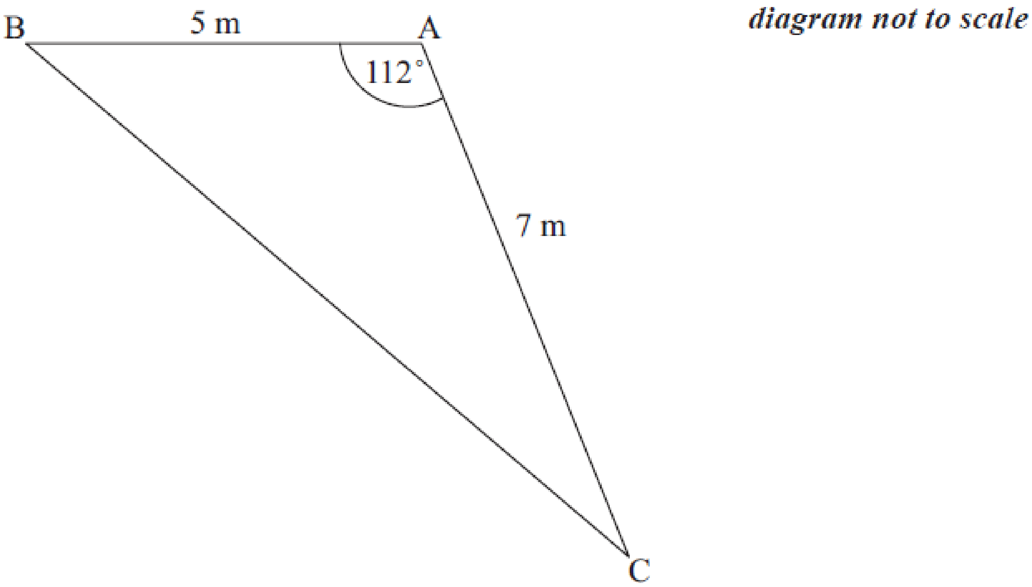
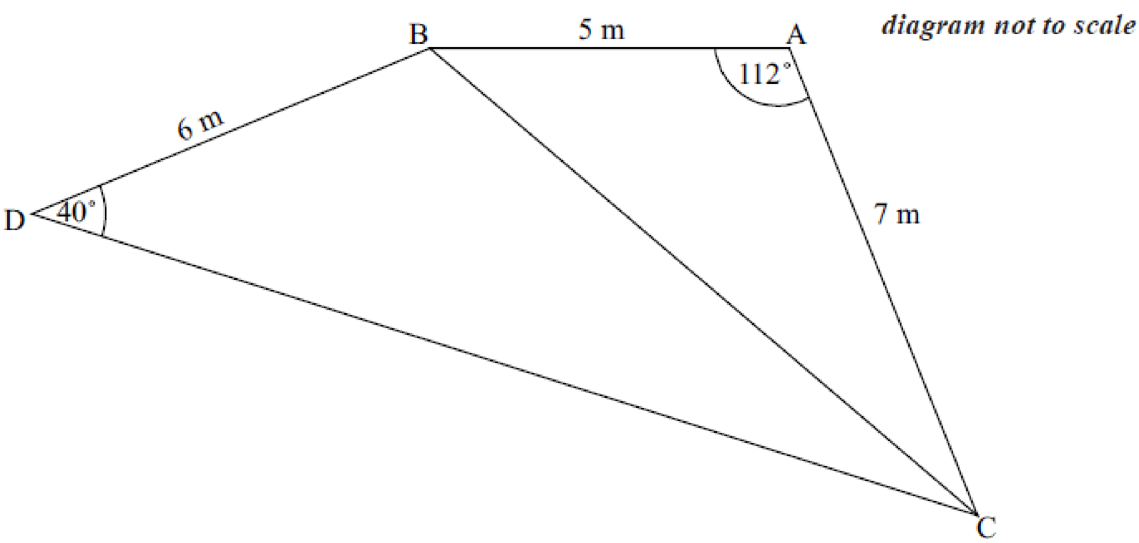
A contractor is building a house. He first marks out three points A , B and C on the ground such that AB = 5 m , AC = 7 m and angle BAC = 112 °.



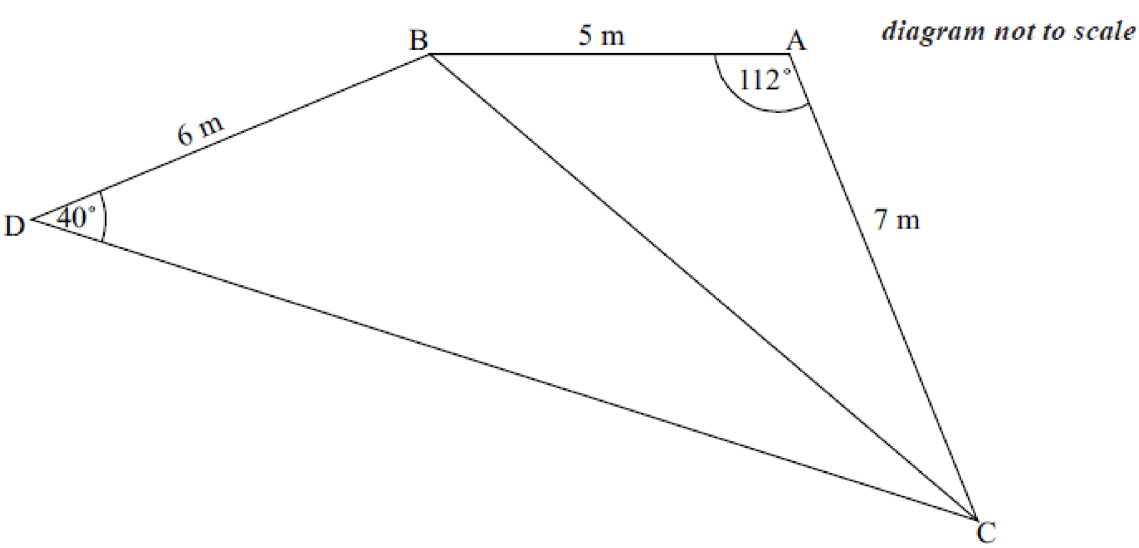
a. Find the length of BC. [3]

b. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [4]



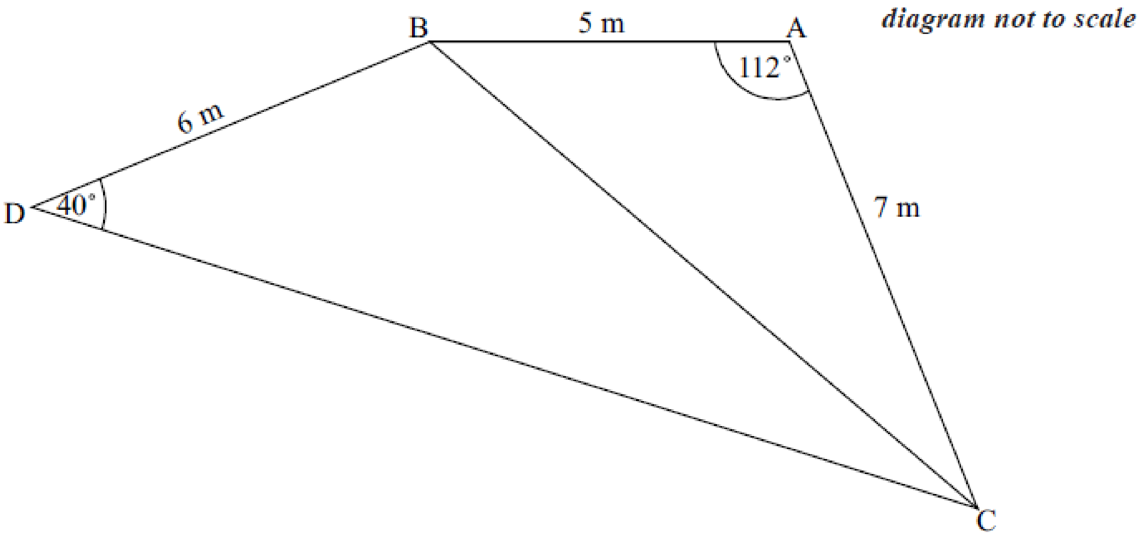
Find the size of angle DBC .

c. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [4]



Find the area of the quadrilateral ABDC.

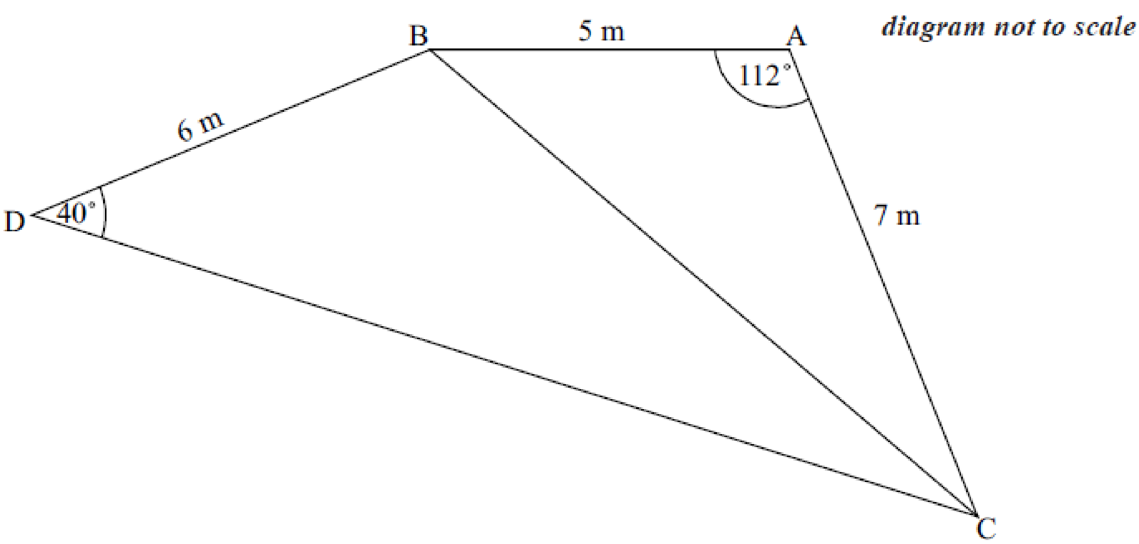
d. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [3]



The contractor digs up and removes the soil under the quadrilateral ABDC to a depth of 50 cm for the foundation of the house.

Find the volume of the soil removed. Give your answer in $\mathbf{m^3}$.

e. He next marks a fourth point, D, on the ground at a distance of 6 m from B , such that angle BDC is 40° . [5]



The contractor digs up and removes the soil under the quadrilateral ABDC to a depth of 50 cm for the foundation of the house.

To transport the soil removed, the contractor uses cylindrical drums with a diameter of 30 cm and a height of 40 cm.

(i) Find the volume of a drum. Give your answer in $\mathbf{m^3}$.

(ii) Find the minimum number of drums required to transport the soil removed.

Markscheme

a. ***Units are required in part (c) only.***

$BC^2 = 5^2 + 7^2 - 2(5)(7)\cos 112°$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution in cosine formula, ***(A1)*** for correct substitutions.

BC = 10.0 (m) (10.0111...) ***(A1)(G2)***

Note: If radians are used, award at most ***(M1)(A1)(A0)***.

[3 marks]

b. ***Units are required in part (c) only.***

$\frac{\sin 40°}{10.0111...} = \frac{\sin \mathrm{D\hat{C}B}}{6}$ ***(M1)(A1)*(ft)**

Notes: Award ***(M1)*** for substitution in sine formula, ***(A1)*(ft)** for their correct substitutions. Follow through from their part (a).

$\mathrm{D\hat{C}B}$ = 22.7° (22.6589...) ***(A1)*(ft)**

Notes: Award ***(A2)*** for 22.7° seen without working. Use of radians results in unrealistic answer. Award a maximum of ***(M1)(A1)*(ft)*(A0)*(ft)**. Follow through from their part (a).

$\mathrm{D\hat{C}B}$ = 117° (117.341...) ***(A1)*(ft)*(G3)***

Notes: Do not penalize if use of radians was already penalized in part (a). Follow through from their answer to part (a).

OR

From use of cosine formula

DC = 13.8(m) (13.8346…) ***(A1)*(ft)**

Note: Follow through from their answer to part (a).

$\frac{\sin \alpha}{13.8346...} = \frac{\sin 40°}{10.0111...}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the correct sine formula.

α = 62.7° (62.6589) ***(A1)*(ft)**

Note: Accept 62.5° from use of 3sf.

$\mathrm{D\hat{B}C}$ = 117(117.341...) ***(A1)*(ft)**

Note: Follow through from their part (a). Use of radians results in unrealistic answer, award a maximum of ***(A1)(M1)(A0)(A0)***.

[4 marks]

c. ***Units are required in part (c) only.***

$\mathrm{ABDC} = \frac{1}{2}(5)(7)\sin 112° + \frac{1}{2}(6)(10.0111...)\sin 117.341...°$ ***(M1)(A1)*(ft)*(M1)*N**

Note: Award ***(M1)*** for substitution in both **triangle** area formulae, ***(A1)*(ft)** for their correct substitutions, ***(M1)*** for seen or implied addition of their two **triangle** areas. Follow through from their answer to part (a) and (b).

= 42.9 m^2 (42.9039...) ***(A1)*(ft)*(G3)***

Notes: Answer is 42.9 m^2 *i.e.* **the units are required** for the final ***(A1)*(ft)** to be awarded. Accept 43.0 m^2 from using 3sf answers to parts (a) and (b). Do not penalize if use of radians was previously penalized.

[4 marks]

d. ***Units are required in part (c) only.***

42.9039... × 0.5 ***(M1)(M1)***

Note: Award ***(M1)*** for 0.5 seen (or equivalent), ***(M1)*** for multiplication of their answer in part (c) with their value for depth.

= 21.5 (m^3) (21.4519...) ***(A1)*(ft)*(G3)***

Note: Follow through from their part (c) only if working is seen. Do not penalize if use of radians was previously penalized. Award at most ***(A0)(M1)(A0)*(ft)** for multiplying by 50.

[3 marks]

e. ***Units are required in part (c) only.***

(i) $\pi(0.15)^2(0.4)$ ***(M1)(A1)***

OR

$\pi \times 15^2 \times 40$ (28274.3...) ***(M1)(A1)***

Notes: Award ***(M1)*** for substitution in the correct volume formula. ***(A1)*** for correct substitutions.

= 0.0283 (m^3) (0.0282743..., 0.09π)

(ii) $\frac{21.4519...}{0.0282743...}$ ***(M1)***

Note: Award ***(M1)*** for correct division of their volumes.

= 759 ***(A1)*(ft)*(G2)***

Notes: Follow through from their parts (d) and (e)(i). Accept 760 from use of 3sf answers. Answer must be a positive integer for the final ***(A1)*(ft)** mark to be awarded.

[5 marks]

Examiners report

a. The responses to this question showed appropriate use of sine and cosine formulae for the most part. A few students still used the Pythagorean formula incorrectly, although the given triangles were not right ones. There was an occasional use of GDC set to radians, and very few students lost marks for giving their answers in radians. In part (d), converting from cm^3 to m^3 was largely problematic for the great majority of students. Part (e) also was difficult for some students, as it requires some interpretation before the volume formula is used.

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Francesca is a chef in a restaurant. She cooks eight chickens and records their masses and cooking times. The mass m of each chicken, in kg, and its cooking time t, in minutes, are shown in the following table.

Mass m (kg)	Cooking time t (minutes)
1.5	62
1.6	75
1.8	82
1.9	83
2.0	86
2.1	87
2.1	91
2.3	98

a. Draw a scatter diagram to show the relationship between the mass of a chicken and its cooking time. Use 2 cm to represent 0.5 kg on the horizontal axis and 1 cm to represent 10 minutes on the vertical axis. [4]

b. Write down for this set of data [2]

(i) the mean mass, $\bar{m}$;

(ii) the mean cooking time, $\bar{t}$.

c. Label the point $\mathrm{M}(\bar{m}, \bar{t}\,)$ on the scatter diagram. [1]

d. Draw the line of best fit on the scatter diagram. [2]

e. Using your line of best fit, estimate the cooking time, in minutes, for a 1.7 kg chicken. [2]

f. Write down the Pearson's product–moment correlation coefficient, *r* . [2]

g. Using your value for *r* , comment on the correlation. [2]

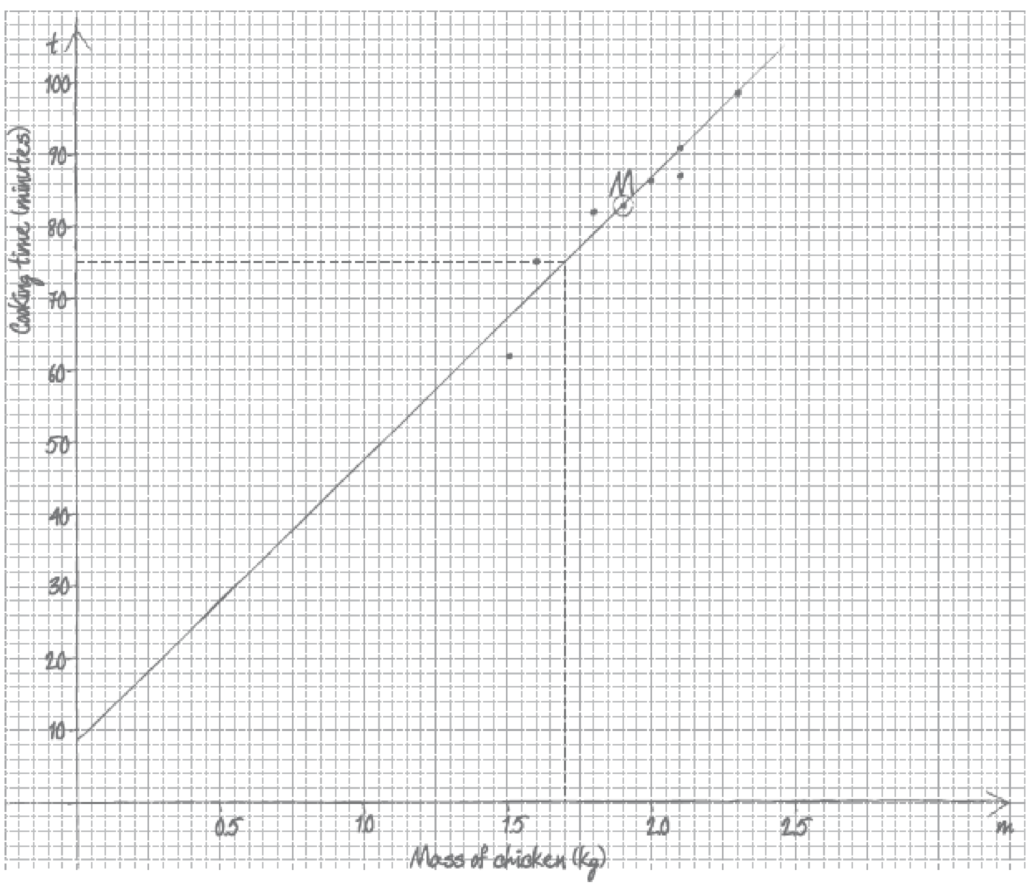
h. The cooking time of an additional 2.0 kg chicken is recorded. If the mass and cooking time of this chicken is included in the data, the correlation is weak. [2]

(i) Explain how the cooking time of this additional chicken might differ from that of the other eight chickens.

(ii) Explain how a new line of best fit might differ from that drawn in part (d).

Markscheme

a.



(A1) *for correct scales and labels (mass or m on the horizontals axis, time or t on the vertical axis)*

(A3) *for 7 or 8 correctly placed data points*

(A2) *for 5 or 6 correctly placed data points*

(A1) *for 3 or 4 correctly placed data points,* ***(A0)*** *otherwise.* ***(A4)***

Note: If axes reversed award at most ***(A0)(A3)*****(ft)**. If graph paper not used, award at most ***(A1)(A0)***.

b. (i) 1.91 (kg) (1.9125 kg) ***(G1)***

(ii) 83 (minutes) ***(G1)***

c. Their mean point labelled. ***(A1)*(ft)**

Note: Follow through from part (b). Accept any clear indication of the mean point. For example: circle around point, (m, t), M , etc.

d. Line of best fit drawn on scatter diagram. ***(A1)*(ft)*(A1)*(ft)**

Notes:Award ***(A1)*(ft)** for straight line through their mean point, ***(A1)*(ft)** for line of best fit with intercept 9(±2) . The second ***(A1)*(ft)** can be awarded even if the line does not reach the t-axis but, if extended, the t-intercept is correct.

e. 75 ***(M1)(A1)*(ft)*(G2)***

Notes: Accept 74.77 from the regression line equation. Award ***(M1)*** for indication of the use of their graph to get an estimate **OR** for correct substitution of 1.7 in the correct regression line equation $t = 38.5m + 9.32$.

f. 0.960 (0.959614...) ***(G2)***

Note: Award ***(G0)(G1)*(ft)** for 0.95, 0.959

g. Strong and positive ***(A1)*(ft)*(A1)*(ft)**

Note: Follow through from their correlation coefficient in part (f).

h. (i) Cooking time is much larger (or smaller) than the other eight ***(A1)***

(ii) The gradient of the new line of best fit will be larger (or smaller) ***(A1)***

Note: Some acceptable explanations may include but are not limited to:

The line of best fit may be further away from the plotted points
It may be steeper than the previous line (as the mean would change)
The t-intercept of the new line is smaller (larger)

Do not accept vague explanations, like:

The new line would vary
It would not go through all points
It would not fit the patterns
The line may be slightly tilted

Examiners report

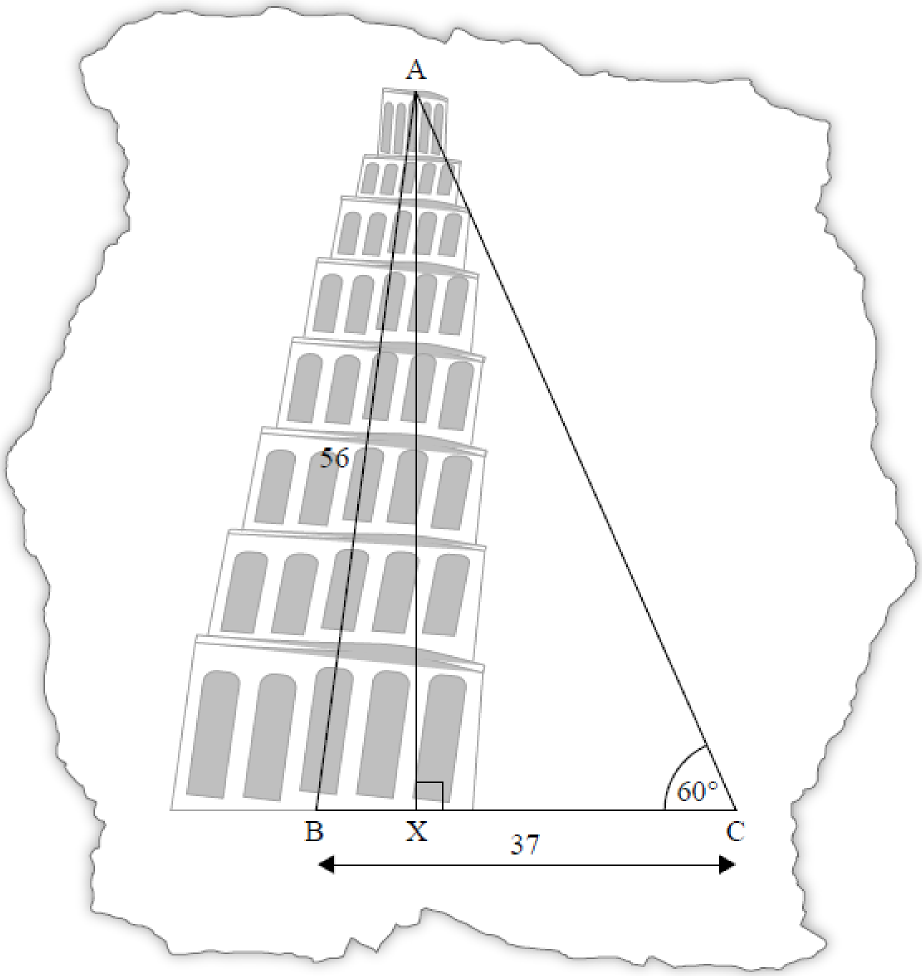
a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]
f. [N/A]
g. [N/A]
h. [N/A]

The Tower of Pisa is well known worldwide for how it leans.

Giovanni visits the Tower and wants to investigate how much it is leaning. He draws a diagram showing a non-right triangle, ABC.

On Giovanni’s diagram the length of AB is 56 m, the length of BC is 37 m, and angle ACB is 60°. AX is the perpendicular height from A to BC.

diagram not to scale



Giovanni’s tourist guidebook says that the actual horizontal displacement of the Tower, BX, is 3.9 metres.

a.i. Use Giovanni’s diagram to show that angle ABC, the angle at which the Tower is leaning relative to the horizontal, is 85° to the nearest degree. [5]

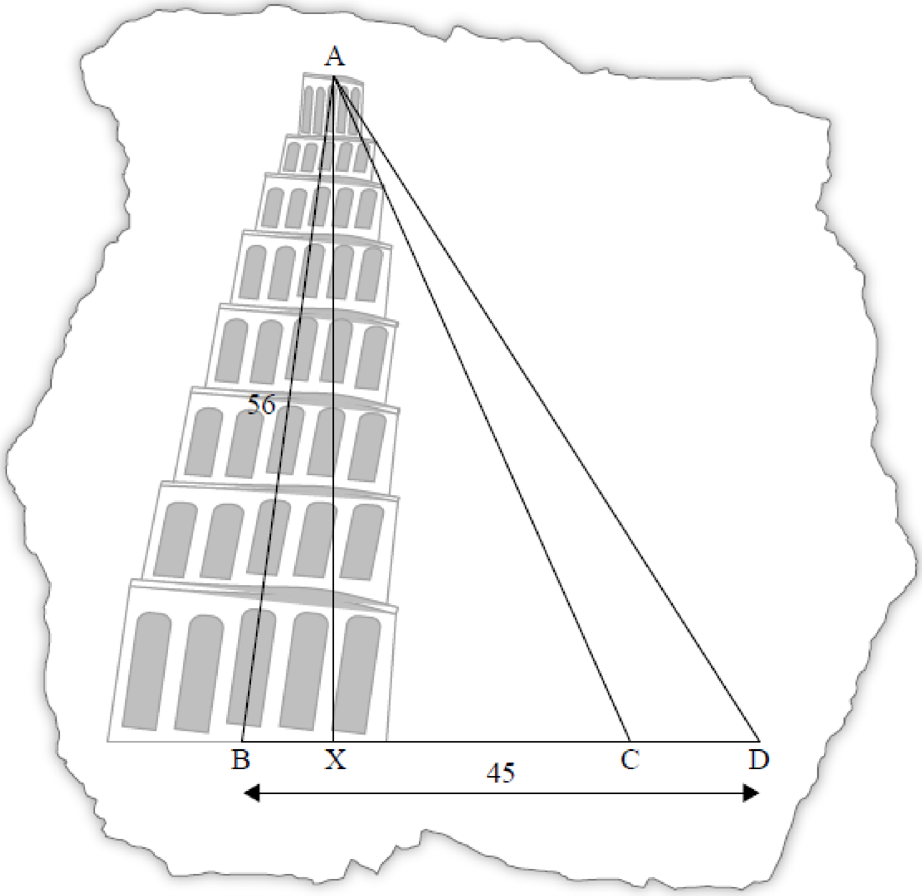
a.ii. Use Giovanni's diagram to calculate the length of AX. [2]

a.iii Use Giovanni's diagram to find the length of BX, the horizontal displacement of the Tower. [2]

b. Find the percentage error on Giovanni’s diagram. [2]

c. Giovanni adds a point D to his diagram, such that BD = 45 m, and another triangle is formed. [3]

diagram not to scale



Find the angle of elevation of A from D.

Markscheme

a.i. $\frac{\sin \mathrm{BAC}}{37} = \frac{\sin 60}{56}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substituting the sine rule formula, ***(A1)*** for correct substitution.

angle $\mathrm{B}\hat{\mathrm{A}}\mathrm{C}$ = 34.9034…° ***(A1)***

Note: Award ***(A0)*** if unrounded answer does not round to 35. Award ***(G2)*** if 34.9034… seen without working.

angle $\mathrm{A}\hat{\mathrm{B}}\mathrm{C}$ = 180 − (34.9034… + 60) ***(M1)***

Note: Award ***(M1)*** for subtracting their angle BAC + 60 from 180.

85.0965…° ***(A1)***

85° ***(AG)***

Note: Both the unrounded and rounded value must be seen for the final ***(A1)*** to be awarded. If the candidate rounds 34.9034...° to 35° while substituting to find angle $\mathrm{A}\hat{\mathrm{B}}\mathrm{C}$, the final ***(A1)*** can be awarded but **only** if both 34.9034...° and 35° are seen.
If 85 is used as part of the workings, award at most ***(M1)(A0)(A0)(M0)(A0)(AG)***. This is the reverse process and not accepted.

a.ii. sin 85… × 56 ***(M1)***

= 55.8 (55.7869…) (m) ***(A1)(G2)***

Note: Award ***(M1)*** for correct substitution in trigonometric ratio.

a.iii. $\sqrt{56^2 - 55.7869\ldots^2}$ ***(M1)***

Note: Award ***(M1)*** for correct substitution in the Pythagoras theorem formula. Follow through from part (a)(ii).

OR

cos(85) × 56 ***(M1)***

Note: Award ***(M1)*** for correct substitution in trigonometric ratio.

= 4.88 (4.88072…) (m) ***(A1)*****(ft)*****(G2)***

Note: Accept 4.73 (4.72863…) (m) from using their 3 s.f answer. Accept equivalent methods.

[2 marks]

b. $\left|\frac{4.88-3.9}{3.9}\right| \times 100$ ***(M1)***

Note: Award ***(M1)*** for correct substitution into the percentage error formula.

= 25.1 (25.1282) (%) ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (a)(iii).

[2 marks]

c. $\tan^{-1}\left(\frac{55.7869\ldots}{40.11927\ldots}\right)$ ***(A1)*****(ft)*****(M1)***

Note: Award ***(A1)*****(ft)** for their 40.11927… seen. Award ***(M1)*** for correct substitution into trigonometric ratio.

OR

$(37 - 4.88072\ldots)^2 + 55.7869\ldots^2$

(AC =) 64.3725…

$64.3726\ldots^2 + 8^2 - 2 \times 8 \times 64.3726\ldots \times \cos 120$

(AD =) 68.7226…

$\frac{\sin 120}{68.7226\ldots} = \frac{\sin \mathrm{A}\hat{\mathrm{D}}\mathrm{C}}{64.3725\ldots}$ ***(A1)*****(ft)*****(M1)***

Note: Award ***(A1)*****(ft)** for their correct values seen, ***(M1)*** for correct substitution into the sine formula.

= 54.3° (54.2781…°) ***(A1)*****(ft)*****(G2)***

Note: Follow through from part (a). Accept equivalent methods.

[3 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
a.iii. [N/A]
b. [N/A]
c. [N/A]